Probability and Statistics

Hypergeometric Distribution Homework

1. There are 50 cars at a car dealership, 20 of which are Fords and 30 of which are Chevrolets. Consider a random sample of 15 cars, and let $X$ be the number of Fords in the sample.

   (a) Calculate $P(X = 7)$.
   (b) Find the expected value, variance, and standard deviation of $X$.
   (c) Why did you use the hypergeometric distribution on this problem?

2. Let $X$ be the number of aces in a five-card poker hand.

   (a) Calculate $P(X = 2)$.
   (b) Find the expected value, variance, and standard deviation of $X$.

3. Bonus: In a clinical trial, 100 volunteers without flu were randomly divided into two groups of 50. The first group (the treatment group) was given a flu vaccine, and the second group (the control group) was given a placebo\(^1\). Later, during flu season, it was found that 2 people from the treatment group had contracted the flu, and 10 people from the control group had contracted the flu.

   (a) Does it seem like the flu vaccine worked?
   (b) Here’s a method to investigate this quantitatively. Assume that the vaccine has no medical benefit. Then 12 people were destined to get flu, and 88 people were destined to not get flu. The difference between the treatment and control groups is just due to chance. Is this likely?

   To calculate this, think of the treatment group as a random sample without replacement from a population with 100 people, 12 of which have flu. Let $X$ be the number of people in the treatment group with flu, and calculate $P(X \leq 2)$.

   (c) Based on your answer to part (3b), does it seem likely that the vaccine has no medical benefit?

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\(^1\)A placebo is a fake medicine given to the control group in a clinical trial. Its purpose is to control for the psychological effect of receiving medication. You might want to read about the “placebo effect” for more info on this topic.