Calculus II Lab 1

1. Calculate the following using Mathematica. Express each answer as an exact value (such as \( \frac{1}{2} \) or \( \sqrt{2} \)) and as a decimal number (such as 0.5 or 1.41421).

   (a) \( \frac{1 + \frac{1}{2} - \frac{2}{3} \cdot \frac{5}{7}}{6^{13}(23) + 7} \).

   (b) \( \sin\left(\frac{\pi}{3}\right) \).

   (c) \( \ln(5) \).

   (d) \( \log_{10}(5) \).

   (e) \( \sqrt[3]{91} \).

   (f) \( e^8 \).

2. Perform the following algebraic tasks in Mathematica.

   (a) Define \( x = 17 + 61\sqrt{37} \), and evaluate \( 5x^3 - 14x^2 + 6x + 9 \).

   (b) Clear the variable \( x \) before proceeding.

   (c) Simplify \( \frac{(a + h)^3 - a^3}{h} \).

   (d) Expand \( (x - 2)^{10} \).

   (e) Factor \( x^4 - 6x^3 + 10x^2 - 6x + 9 \).

   (f) Solve \( 35 - 5x = 7x^2 - x^3 \) for \( x \), providing both exact solutions and decimal approximations.

3. Suppose \( f(x) = xe^{-x} \).

   (a) Define this function in Mathematica, and use it to calculate \( f(0) \) and \( f(1) \). Do these answers look right?

   (b) Plot this function for \( 0 < x < 10 \).

   (c) Find \( f'(x) \) and \( f'(5) \). Is \( f'(5) \) positive or negative? What does this tell you about the graph of \( f \) at \( x = 5 \)?

   (d) Find \( f''(x) \) and \( f''(5) \). Is \( f''(5) \) positive or negative? What does this tell you about the graph of \( f \) at \( x = 5 \)?

   (e) Find \( f^{(100)}(x) \), that is, the 100th derivative of \( f \).

   (f) Find \( \int f(x) \, dx \).

   (g) Calculate the area under the curve of this function from \( x = 2 \) to \( x = 6 \).

   (h) Find all values of \( x \) for which \( f(x) = 0.2 \).

   (i) Confirm that these are solutions by plotting \( f \) and the line \( y = 0.2 \) for \( 0 < x < 10 \).

   (j) Calculate the area between the curves \( y = f(x) \) and \( y = 2 \).