Calculus II Review Three

Please show your work on all problems.

1. Find the arc length of the curve \( y = \ln(x) \), \( 1 \leq x \leq 4 \).

2. Find the area of the surface obtained by rotating the curve \( y = \sin(x) \), \( 0 \leq x \leq \pi \) about the \( x \)-axis.

3. Find the centroid of the region bound by the curves \( y = x^2 \) and \( y = 25 \).

4. A lake is held back by a dam whose shape is an isosceles triangle with a width of 20 m at the top and a height of 30 m (see diagram). Assuming the water level is at the top of the dam, how much force is exerted on the dam by the water?

![Diagram of a dam](https://example.com/dam-diagram)

5. Consider the parametric curve \( x = 2t^3 + 3t + 5, y = t^3 + 7t + 1 \), \( 0 \leq t \leq 5 \).
   
   (a) Find the slope of the tangent line to this curve at \( t = 4 \).
   
   (b) Find the area under the curve.
   
   (c) Find the arc length of the curve.
   
   (d) Find the surface area obtained by rotating the curve about the \( x \)-axis.

6. Consider the polar curve \( r = 2 + \sin(3\theta) \), \( 0 \leq \theta \leq 2\pi \).
   
   (a) Find the slope of the tangent line to this curve at \( \theta = \frac{\pi}{4} \).
   
   (b) Find the area enclosed by this curve.
   
   (c) Find the arc length of the curve.
Answers

1. 3.34
2. 14.42
3. (0, 15)
4. \(2.94 \times 10^7\) N
5. (a) \(\frac{5}{9}\)
   (b) 23183.75
   (c) 311.10
   (d) 166978
6. (a) \(-0.121\)
   (b) 14.14
   (c) 18.15