Math 505 Homework 1

1. Suppose $X$ has a binomial distribution with parameters $n = 20$ and $p = 0.8$.
   (a) Find the p.m.f. for $X$.
   (b) Find $P[X = 10]$.
   (c) Find $E[X]$, $\text{Var}[X]$, and $\sigma_X$.

2. If $\theta > 0$, an exponential distribution with parameter $\theta$ is given by the p.d.f.
   \[ f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta}, & 0 \leq x < \infty \\ 0, & \text{otherwise} \end{cases} \]
   (a) If $X$ has an exponential distribution with parameter $\theta$, show that $E[X] = \theta$, and $\text{Var}[X] = \theta^2$.
   (b) If $X$ has an exponential distribution with parameter $\theta = 5$, find $E[X]$, $\text{Var}[X]$, and $P[2 < X < 8]$.

3. Consider a population of women whose heights are normally distributed with mean 64 inches and standard deviation 2 inches. Find the probability that a randomly selected woman from this population has a height between 61 and 65 inches.

4. Let $Z$ be a standard normal random variable.
   (a) Find the approximate value of $c$, where $P[Z < c] = 0.84$.
   (b) Find the approximate value of $z_{0.07}$.

5. Let $T$ have a $t$-distribution with 18 degrees of freedom.
   (a) Find the approximate value of $c$, where $P[T < c] = 0.75$.
   (b) If $\alpha = 0.05$, find the approximate value of $t_{\alpha/2}(18)$.