Properties of Logarithm Functions

These properties follow from the definition of a logarithm function as the inverse of an exponential function, and from the exponent rules.

1. \( \log_b 1 = 0 \) [zero power rule: \( b^0 = 1 \)]
2. \( \log_b b = 1 \) [\( b^1 = b \)]
3. \( \log_b b^x = x \) [composition of a function with its inverse is the identity function]
4. \( b^{\log_b x} = x \) [composition of a function with its inverse is the identity function]
5. \( \log_b M = \log_b N \) if and only if \( M = N \) [log functions are one-to-one].
6. \( \log_b (M \cdot N) = \log_b M + \log_b N \) [from the addition of exponents rule: \( a^m a^n = a^{m+n} \)]
7. \( \log_b \left( \frac{M}{N} \right) = \log_b M - \log_b N \) [from the subtraction of exponents rule: \( \frac{a^m}{a^n} = a^{m-n} \)]
8. \( \log_b M^p = p \log_b M \) [from the power rule: \( (a^m)^n = a^{mn} \)]