Calculus I Lab 1

1. Calculate the following using Mathematica. Express each answer as an exact value (such as $\frac{1}{2}$ or $\sqrt{2}$) and as a decimal number (such as 0.5 or 1.41421).

(a) \(\frac{1 + \frac{1}{2} - \frac{2}{3} \cdot \frac{5}{7}}{\frac{6}{\sqrt{3}}(23) + 7}\).

(b) \(\sin(\frac{\pi}{3})\).

(c) \(\ln(5)\).

(d) \(\log_{10}(5)\).

(e) \(\sqrt[3]{91}\).

(f) \(e^8\).

2. Perform the following algebraic tasks in Mathematica.

(a) Define \(x = 17 + 61\sqrt{37}\), and evaluate \(5x^3 - 14x^2 + 6x + 9\).

(b) Simplify \(\frac{(a + h)^3 - a^3}{h}\).

(c) Expand \((x - 2)^{10}\).

(d) Factor \(x^4 - 6x^3 + 10x^2 - 6x + 9\).

(e) Solve \(35 - 5x = 7x^2 - x^3\) for \(x\), providing both exact solutions and decimal approximations.

3. In this problem, you will investigate the behavior of \(f(x) = \frac{\sin(x)}{x}\) near \(x = 0\).

(a) Is \(f(0)\) defined?

(b) Calculate \(f(x)\) for the following values of \(x\): 0.1, 0.01, 0.001, and 0.0001. Also evaluate \(f(x)\) at the negatives of these values. Based on this information, estimate \(\lim_{x \to 0} \frac{\sin(x)}{x}\).

(c) Plot the graph of \(f\) using the window \(-2 \leq x \leq 2\), \(0 \leq y \leq 2\). Based on this plot, estimate \(\lim_{x \to 0} \frac{\sin(x)}{x}\).

(d) Calculate \(\lim_{x \to 0} \frac{\sin(x)}{x}\) directly using Mathematica’s \textit{Limit} command.

(e) How could you use the \textit{Manipulate} command to investigate \(\lim_{x \to 0} \frac{\sin(x)}{x}\)?

4. Explore Mathematica’s capabilities further. For instance, you could pose your own problems and figure out how to solve them or think of creative uses of the \textit{Manipulate} command.