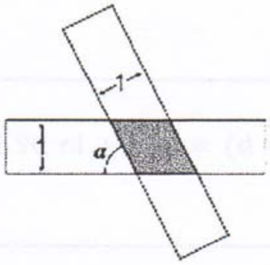
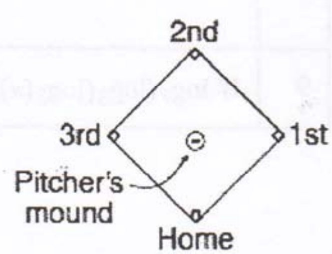
Calculus/PreCalculus

Problems:

1	The lengths of the sides of a cyclic quadrilateral are 1, 9, 9, and 6, as shown below. Find *cos B*.
	cosB1
2	What is the value of $\log_3 16 \bullet \log_2 27$?
3	Find n so that $\frac{1}{1+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\cdots+\frac{1}{\sqrt{2n-1}+\sqrt{2n+1}}=100.$
4	If $\log_8 3 = p$ and $\log_3 5 = q$, express $\log_{10} 5$ as a function of p and q.
5	Find the sum of the arithmetic series (-300) + (-297) + (-294) + . . . + (306) + (309).
6	An airplane was supposed to cover the distance of 2900 km. However, after covering 1700 km, it had to land and wait on the ground for 1 hour and 30 minutes. After it took off again, its average speed was 50 km/h less than before. Find the original average speed of the plane if it is known that it completed the flight 5 hours after departure.
7	do any positive real numbers *a* and *b* exist such that ln(*a* + *b*) = ln *a* + ln *b*? If so, what are they?
8	In how many points do the curves $x = y^2$ and $x^2 = (y - 1)^2$ intersect?
9	If $\log_7(\log_3(\log_2(x))) = 0$, then express $x^{-1/2}$ as a real number in radical form.

10	A sequence of three real numbers begins with 9 and forms an arithmetic progression. 2 is added to the second term, 20 to the third term. The three new numbers form a geometric progression. What is the smallest possible value for the third term of the new progression?
11	The rule of 70 says that you can estimate the time (in years) needed to double the amount in a bank account paying *r* percent annual interest by dividing 70 by *r*. Explain why this method works for *continuous* compounding. Hint: Start by solving a doubling time equation for continuous compounding using logarithms.
12	Find the real value of x such that $3 + 3\log_3(x^3 + 1) = 3^2$.
13	N and Y are positive integers. $N \neq Y$. If $N \times Y \div 2 = N + Y$, what is the sum $N + Y$?
14	Al offers to sell Betty his bicycle for $100. Betty offers $50. Al comes down to $75, to which Betty offers $62.50. They continue haggling in this way, each time suggesting the average of the previous two amounts. On what amount do they converge?
15	Two strips of width 1 overlap at an angle of a, as shown. Express the shaded area in terms of a. 
16	A baseball diamond is a square with side length of 90 ft. The pitcher's mound is 60 ft. 6 in. from home plate and lies on the line between home plate and second base. How far is the pitcher's mound from first base? 

17	How long is the side of the largest equilateral triangle that can be inscribed in a square with side length 1?
18	Compute the sum of all ten-digit numbers. Can you determine the sum of all *n*-digit numbers?
19	If i represents the imaginary unit, find real values for a and b such that $(1+i)^{13} = a + bi$.
20	If x , y, and z are three different numbers for which the ordered triples x, y, z and x^3, y^3, z^3 are in arithmetic progression, find y.

21. *Shaded Squares:* Assume that each shaded square represents part of the area of the larger square bordering two of its adjacent sides and that the shading continues indefinitely in the indicated manner.

- How much of the total area of the square is shaded? Thoroughly justify your thinking.