Math 5301 Homework 4

If you are rusty on any of these topics and need additional practice, you can do p. 32 (1,4) and p. 63 (1–4).

1. How many 5-card poker hands are there that contain no Kings, Queens, or Jacks? (Assume a standard deck of 52 cards, which contains 4 Kings, 4 Queens, and 4 Jacks.)

2. In a group of 100 people, 12 have a certain disease. Let X be the number of people with this disease in a random sample of size 50 without replacement from this group. Find the following:

   (a) $P(X \leq 2)$

   (b) $E(X)$, $\mathrm{Var}(X)$, and σ_X.

3. Consider the sample space $S = \{1, 2, 3, 4\}$, where all the outcomes are equilikely, and define the events $A = \{1, 2\}$, $B = \{1, 3\}$, and $C = \{1, 4\}$.

   (a) Are A and B independent?

   (b) Are A and C independent?

   (c) Are B and C independent?

   (d) Are A, B, and C independent?

4. Suppose 20 customers take out loans from a bank, and there is a 15% chance that each customer will default on his or her loan. Assume the customers are statistically independent, and let X be the number of customers who default. Find the following:

   (a) $P(X = 4)$

   (b) $E(X)$, $\mathrm{Var}(X)$, and σ_X.

   (c) $P(2 \leq X \leq 6)$ (use Table A3 in the book)

5. Gas mileages for a certain type of car are approximately normally distributed with mean 35.4 mpg and standard deviation 2.6 mpg. Find the probability that a randomly selected car of this type has a gas mileage between 32 and 37 mpg using Table A1 in the book.

6. Do p. 63 (5), using the normal approximation to the binomial distribution.

7. A certain type of scratch-off lottery ticket costs \$1. For a single ticket, there is an 80% chance of winning nothing, a 15% chance of winning \$2, and a 5% chance of winning \$10.

   (a) Let X be the *net winnings* from a single ticket, that is, $X = \text{prize} - 1$. Find $E(X)$ and σ_X.

   (b) For 1000 of these tickets that are statistically independent, find the approximate distribution of the total net winnings,
   $$Y = X_1 + \cdots + X_{1000}.$$

   (c) Find the approximate probability of at least breaking even, that is, $P(Y \geq 0)$.