## Math 5366 Homework 32

1. Solve the following system of equations using SAS.

$$
\begin{aligned}
x+z & =6 \\
2 y-4 z & =-8 \\
3 x+7 y & =1 .
\end{aligned}
$$

2. Write a PROC IML function that accepts the lengths $a$ and $b$ of the legs of a right triangle and returns the length of the hypotenuse. Test it with the inputs $a=3$ and $b=4$.
3. Assume $\mu \in \mathbb{R}^{1 \times p}, \Sigma^{1 / 2} \in \mathbb{R}^{p \times p}$ is positive definite, $Z$ is an $n \times p$ random matrix with standard normal entries, and $\mathbb{1}_{n \times 1} \in \mathbb{R}^{n \times 1}$ is a column vector of ones. Then

$$
X=\mathbb{1}_{n \times 1} \mu+Z \Sigma^{1 / 2}
$$

is a random matrix whose rows are normally distributed with mean vector $\mu$ and covariance matrix $\Sigma$.
(a) Write a PROC IML function that accepts $\mu, \Sigma^{1 / 2}$, and $n$, and returns the randomly generated matrix $X$.
(b) Apply this function to the inputs $\mu=(5,10), \Sigma^{1 / 2}=\left(\begin{array}{l}7 \\ 1 \\ 3\end{array}\right)$, and $n=100,000$. Output the resulting matrix into a SAS data set.
(c) Apply PROC CORR with the COV option to this SAS data set to estimate the covariance matrix $\Sigma$, and compare the result to the actual value of $\Sigma$.
4. The file FTIC.csv from Homework 1 contains data for 9,218 first time in college freshman, admitted to TSU between 2004 and 2011. In this problem, we will revisit this data set using SAS instead of R.
(a) Find the retention rate for all students in the data set.
(b) The table below shows TSU's admissions requirements from fall 2012. Note that percentile ranks of 90 to 100 are not listed, because public universities are required by state law to admit anyone in the top $10 \%$ of their class. Also, the minimum score possible on the SAT is 400 , so all students with ranks from 50 to 89 are admitted under this policy.

| Percentile Rank | $1-24$ | $25-49$ | $50-89$ |
| :---: | :---: | :---: | :---: |
| SAT Requirement | 1030 | 950 | 400 |

Verify that there are 547 students in our data set who were admitted, even though they don't satisfy these requirements. Also, verify that the retention rate among students who do satisfy the requirements is $67.5 \%$.
(c) In part (b), we effectively have a vector of rank thresholds $(25,50)$ and a vector of SAT thresholds (1030, 950, 400). Write a PROC IML function called that accepts the rank thresholds and SAT thresholds and returns the enrollment loss and new retention. ${ }^{1}$

[^0](d) In fall 2012, the Office of Enrollment Management recommended no longer admitting students with percentile ranks below 33 , which is equivalent to the following requirements.

| Percentile Rank | $1-32$ | $33-49$ | $50-89$ |
| :---: | :---: | :---: | :---: |
| SAT Requirement | 1610 | 950 | 400 |

Find the enrollment loss and new retention for this policy.
5. The following is a simple Monte Carlo method for estimating $\pi$.

- Let $X_{i}$ and $Y_{i}$ be uniformly distributed on $[0,1]$, for $i=1, \ldots, n$.
- Let $m$ be the number of times that $X_{i}^{2}+Y_{i}^{2}<1$.
- The estimate for $\pi$ is $4 \frac{m}{n}$.
(a) Why does this method for estimating $\pi$ work?
(b) Obtain estimates for $\pi$ using this method and $n=10^{3}, 10^{4}, 10^{5}, 10^{6}$. Compare your estimates to the actual decimal expansion of $\pi$.


[^0]:    ${ }^{1}$ There are a variety of ways to implement this. Feel free to use the method you are most comfortable with.

