Calculus I Lab 2

1. Suppose $f(x) = x^{10}$.

   (a) Find $f'(x)$
   (b) Find $f'(4)$
   (c) Find $f''(x)$
   (d) Find $f'''(x)$
   (e) Find $f^{(8)}(x)$
   (f) Find $f^{(8)}(-2)$

2. Suppose the position of a particle at time t is $s(t) = \sec(t^3)$.

   (a) Find the velocity and acceleration of the particle at time t.
   (b) Find the velocity and acceleration of the particle at time $t = 3$.

3. Let $g(x) = \sqrt{x}$.

   (a) Find the tangent line to the graph of g at $(4, 2)$.
   (b) In Mathematica, define $L(x)$ to be the equation of this tangent line.
   (c) Graph g and its tangent line on the same graph, over the interval $0 \le x \le 12$.
   (d) Repeat step (3c) using the interval $3 \le x \le 5$.
   (e) Repeat step (3c) using the interval $3.9 \le x \le 4.1$.
   (f) As you zoom in on these two graphs, what do you notice?
   (g) Calculate $g(x)$ and $L(x)$, for $x = 8$, 6, 5, 4.5, 4.1, and 4.01.
   (h) In a book, you find the statement, "Whenever x is close to 4, $\sqrt{x} \approx \frac{1}{4}x + 1$". Do you agree or disagree? Why? How did the author figure this out?

4. Differentiate
$$\cos\left(\frac{e^x}{\ln(x) + 4x^5}\right)\sqrt{\csc(x) + \cot(x)}.$$