

Probability and Statistics Exam Two

Name: _____

Please show your work on all problems. You may only use your calculator's statistical functions or the attached table for problem 2 part c. You may use your calculator to perform arithmetic for all problems.

1. **(30 pts)** In a shipment of 50 items, 10 are defective. Suppose 8 items are selected from this shipment at random without replacement, and let X be the number of defective items in the sample.

(a) **(5 pts)** Find the probability mass function for X .

(b) **(10 pts)** Find the probability that at least 2 items in the sample are defective.

(c) **(10 pts)** Find the expected value of X .

(d) **(5 pts)** What type of distribution does X have?

2. **(40 pts)** A multiple choice exam has 15 questions, and there are 4 possible answers for each question. Suppose a student guesses randomly and independently on these questions, and let X be the number of questions answered correctly by that student.

(a) **(5 pts)** Find the probability mass function for X .

(b) **(10 pts)** Find the probability that exactly 2 of the questions are answered correctly.

(c) **(10 pts)** Find the probability that at least 10 of these questions are answered correctly. You may use your calculator's statistical functions or the attached table to do this part of the problem.

(d) **(10 pts)** Find the expected value, variance, and standard deviation of X .

(e) **(5 pts)** What type of distribution does X have?

3. **(25 pts)** Let X be a random variable with p.m.f. $f(x) = cx^3, x = 1, 2, 3$, where c is a constant.

(a) **(10 pts)** Find c , and calculate $P(X = 2)$.

(b) **(10 pts)** Find $E(X)$, $\text{Var}(X)$, and σ_X .

4. **70 pts.** Customers enter a store according to a Poisson process at an average rate of 12 per hour, or $\frac{1}{5}$ per minute. Let X be the number of customers entering the store during the first 20 minutes of business.

(a) What type of distribution does X have? What is the probability mass function for X ?

(b) What is the expected value and variance of X ?

(c) Find the probability that the number of customers entering the store during the first 20 minutes of business is at most 2.