## Calculus II Final Exam

Please show your work on all problems.

1. Find the area enclosed by the curves $y=x^{2}$ and $y=4 x-x^{2}$.
2. Consider the region bound by the curves $y=e^{-x}, y=0, x=1$, and $x=10$. Find the volume of the solid obtained by rotating this region about the $x$-axis using the disk method.
3. An aquarium has a 2 ft by 1 ft rectangular base, and it is 1 ft tall. If it is full of water, how much work is required to pump all of the water out of the aquarium (water weighs 62.5 $\left.\mathrm{lb} / \mathrm{ft}^{3}\right)$ ?
4. Find $\int x^{2} e^{x} d x$ without using a calculator.
5. Does the series $\sum_{n=0}^{\infty} \frac{1}{8^{n}}$ converge? Justify your answer.
6. Does the series $\sum_{n=1}^{\infty} \frac{4 n+5}{7 n+2}$ converge? Justify your answer.
7. Find the arc length of the curve $y=\cos x, 0 \leq x \leq \frac{\pi}{2}$.
8. Find the area of the surface obtained by rotating the curve $y=\sqrt{100-x^{2}},-10 \leq x \leq 10$, about the $x$-axis.
9. Consider the parametric curve $x=5 t^{3}+7 t+1, y=7 t^{2}+9 t+6,0 \leq t \leq 5$. Find the slope of the tangent line to this curve at $t=4$.
10. Find $\int \frac{5 x-1}{x^{2}+x-12} d x$ without using a calculator.
11. Evaluate $\int_{0}^{1} x^{3} \sqrt{1-x^{2}} d x$ without using a calculator.
12. Find the radius and interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(x-5)^{n}}{n}$.
