Probability and Statistics

Moment-Generating Function Homework

1. Suppose X has a binomial distribution with parameters n and p. Then its moment-generating function is
$$M(t) = (1 - p + pe^t)^n.$$
   (a) Use the m.g.f. to show that $E(X) = np$ and $\mathrm{Var}(X) = np(1-p)$.

   (b) **Bonus.** Prove that the formula for the m.g.f. given above is correct. Hint: the binomial theorem says that
$$\sum_{x=0}^{n} \binom{n}{x} a^x b^{n-x} = (a+b)^n.$$

2. Suppose X has a Poisson distribution with parameter λ. Then its moment-generating function is
$$M(t) = e^{\lambda(e^t - 1)}.$$
   (a) Use the m.g.f. to show that $E(X) = \lambda$ and $\mathrm{Var}(X) = \lambda$.

   (b) **Bonus.** Prove that the formula for the m.g.f. given above is correct. Hint: for any real number a,
$$\sum_{x=0}^{\infty} \frac{a^x}{x!} = e^a.$$