

Probability and Statistics II Final Exam Review

Please show your work on all problems. Full use of your calculator is permitted.

1. Let X and Y be random variables with joint p.d.f.

$$f(x, y) = c(x^2 + y^2), \text{ for } 0 < x < 4 \text{ and } 0 < y < 2.$$

Determine the following.

- (a) c
 - (b) $P(X < Y^2)$
2. Let X be a continuous random variable with p.d.f. $f(x) = 3x^2$, $0 < x < 1$. Find the p.d.f. of $Y = 5X^2$.
 3. Suppose that X_1, \dots, X_n are independent random variables, and X_i has a binomial distribution with parameters n_i and p , for $i = 1, \dots, n$. Find the distribution of $X_1 + \dots + X_n$, and justify your answer.
 4. Suppose U_1, U_2, \dots are independent random variables, each having a uniform distribution on the interval $[0, 1]$. If $Y = U_1 + \dots + U_{500}$, approximate $P(245 < Y < 260)$.
 5. Let $\theta > 0$, and consider the p.d.f. $f(x) = \theta x^{\theta-1}$, $0 < x < 1$. Based on a random sample X_1, \dots, X_n from this distribution, find the maximum likelihood estimator for θ .
 6. In a random sample of 1000 machine components, 23 were defective. Find a 95% confidence interval for p , the proportion of machine components in the population that are defective.
 7. Does the sample from the previous problem provide strong evidence that $p < 0.04$? Perform a hypothesis test to support your conclusion.
 8. The yield from a certain agricultural plant is normally distributed with mean μ . For a random sample of 15 of these plants, the sample mean was 54.23 grams, and the standard deviation was 12.78 grams. Find a 95% confidence interval for μ .
 9. Does the sample from the previous problem provide strong evidence that $\mu \neq 50$? Perform a hypothesis test to support your conclusion.