Math 505 Homework 2

1. The *Poisson distribution* with parameter $\lambda > 0$ is given by the p.m.f.

$$f(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!},\ x = 0, 1, 2, \ldots.$$

   Find the maximum likelihood estimator $\hat{\lambda}$ based on a sample $X_1, \ldots, X_n$ from a Poisson distribution.

2. A random sample of 2000 Math SAT scores yields $\overline{X} = 502.37$ and $s = 118.74$.

   (a) Find a 95% confidence interval for the population mean μ.

   (b) Perform the following hypothesis test at the 5% significance level:

$$\mathrm{H}_0 : \mu = 500 \text{ vs. } \mathrm{H} : \mu \neq 500.$$

   (c) Find the p-value of the test from part (b).

3. Suppose $X_1, \ldots, X_n$ is a random sample from a $N(\mu, \sigma^2)$ population.

   (a) Use the fact that

$$T = \frac{\overline{X} - \mu}{s/\sqrt{n}} \sim t(n-1)$$

   to show that the random interval

$$\left(\overline{X} - t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}}, \overline{X} + t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}}\right)$$

   contains μ with probability $1 - \alpha$.

   (b) Let $\mu_0 \in \mathbb{R}$, and consider the testing problem

$$\mathrm{H}_0 : \mu = \mu_0 \text{ vs. } \mathrm{H} : \mu \neq \mu_0.$$

   If we reject H_0 if and only if $|T| \geq t_{\alpha/2}(n-1)$, show that the probability of making a Type I error is at most α.