Probability and Statistics II
MLE Homework

For each problem, find the following:

(a) The likelihood function.
(b) The log-likelihood function.
(c) The maximum likelihood estimator for the unknown parameter.
(d) The numerical value of the MLE if data are given.

1. Beta-particles are emitted by a radioactive sample according to a Poisson process at an average rate of λ per minute. Let X_i be the number of particles emitted during the ith minute of an experiment, where $i = 1, \ldots, n$. What would your estimate for the parameter be if $n = 100$ and $\sum_{i=1}^{100} X_i = 278$?

2. Suppose $\epsilon_1, \ldots, \epsilon_n$ are measurement errors that are normally distributed with mean 0 and variance σ^2. What would your estimate for the parameter be if $n = 5$, $\epsilon_1 = 1.3$, $\epsilon_2 = -2.4$, $\epsilon_3 = 2.1$, $\epsilon_4 = -0.8$, and $\epsilon_5 = -0.3$?

3. Suppose $X_1, \ldots, X_n$ are measurements of the mass of an object. The measurements are normally distributed, with mean equal to μ, the mass of the object, and standard deviation equal to 1 microgram.

4. A telemarketer makes n statistically independent sales calls, where the probability of success on a single call is p. Let X denote the number of successful calls made. Find the MLE for p in terms of X and n. If the telemarketer makes 168 calls of which 12 are successful, estimate p.

5. Consider a weighted coin which, when flipped, lands heads-up with probability p. Suppose the coin is flipped until it lands heads up, and let X be the number of flips required. Note that X has a geometric distribution with parameter p. Find the MLE for p. If 5 flips are required before the coin lands heads-up, estimate p.