1. Bivariate Distributions

1.1. **Integral Limits.** Keep in mind that the integral limits $\int_{-\infty}^{\infty}$ and $\int_a^b$ are restricted to the support of the variable, e.g., if $f(x) = 2x$, $0 \le x \le 1$, then

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx = \int_0^1 x \cdot 2x dx = \tfrac{2}{3}.$$

1.2. **Joint p.d.f. of (X, Y).**

$$P(a \le X \le b, c \le Y \le d) = \int_a^b \int_c^d f(x, y) dy dx.$$

1.3. **Marginal p.d.f. of X.**

$$f_1(x) = \int_{-\infty}^{\infty} f(x, y) dy$$

$$P(a \le X \le b) = \int_a^b f_1(x) dx$$

$$\mu_X = E(X) = \int_{-\infty}^{\infty} x f_1(x) dx$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f_1(x) dx$$

$$\sigma_X^2 = \mathrm{Var}(X) = E(X^2) - E(X)^2$$

1.4. **Marginal p.d.f. of Y.**

$$f_2(y) = \int_{-\infty}^{\infty} f(x, y) dx$$

$$P(a \le Y \le b) = \int_a^b f_2(y) dy$$

$$\mu_Y = E(Y) = \int_{-\infty}^{\infty} y f_2(y) dy$$

$$E(Y^2) = \int_{-\infty}^{\infty} y^2 f_2(y) dy$$

$$\sigma_Y^2 = \mathrm{Var}(Y) = E(Y^2) - E(Y)^2$$

1.5. **Independence, Covariance, and Correlation.**

Definition 1.1. X and Y are *statistically independent* if $f(x, y) = f_1(x) f_2(y)$.

$$E(XY) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy f(x, y) \, dy dx.$$

$$\sigma_{X,Y} = \mathrm{Cov}(X, Y) = E(XY) - E(X)E(Y)$$

$$\rho = \mathrm{Corr}(X, Y) = \frac{\mathrm{Cov}(X, Y)}{\sigma_x \sigma_Y}$$

1.6. **Conditional p.d.f. of X.**

$$g(x|y) = \frac{f(x,y)}{f_2(y)}$$

$$P(a \leq X \leq b \mid Y = y) = \int_a^b g(x|y)dx$$

$$\mu_{x|y} = E(X \mid Y = y) = \int_{-\infty}^{\infty} xg(x|y)dx$$

$$E(X^2 \mid Y = y) = \int_{-\infty}^{\infty} x^2 g(x|y)dx$$

$$\sigma_{x|y}^2 = \mathrm{Var}(X \mid Y = y) = E(X^2 \mid Y = y) - E(X \mid Y = y)^2$$

1.7. **Conditional p.d.f. of Y.**

$$h(y|x) = \frac{f(x,y)}{f_1(x)}$$

$$P(a \leq Y \leq b \mid X = x) = \int_a^b h(y|x)dy$$

$$\mu_{y|x} = E(Y \mid X = x) = \int_{-\infty}^{\infty} yh(y|x)dy$$

$$E(Y^2 \mid X = x) = \int_{-\infty}^{\infty} y^2 h(y|x)dy$$

$$\sigma_{y|x}^2 = \mathrm{Var}(Y \mid X = x) = E(Y^2 \mid X = x) - E(Y \mid X = x)^2$$