Calculus III Review Five

1. Consider the vector field $\mathbf{F}(x, y) = xy^2\mathbf{i} + x^2y\mathbf{j}$ and the curve $\mathbf{r}(t) = \langle t + \sin(\frac{\pi}{2}t), t + \cos(\frac{\pi}{2}t)\rangle$, $0 \leq t \leq 1$.

   (a) Find a function f, such that $\mathbf{F} = \nabla f$.

   (b) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

2. Compute $\int_C y^3\, dx - x^3\, dy$, where C is the circle $x^2 + y^2 = 4$, oriented counterclockwise.

3. Consider the vector field $\mathbf{F}(x, y, z) = x^2z^2\mathbf{i} + y^2z^2\mathbf{j} + xyz\mathbf{k}$. Let S be the part of the paraboloid $z = x^2 + y^2$ lying inside the cylinder $x^2 + y^2 = 4$, oriented upward, and evaluate $\iint_S \text{curl}\, \mathbf{F} \cdot d\mathbf{S}$.

4. Consider the vector field $\mathbf{F}(x, y, z) = xye^z\mathbf{i} + xy^2z^3\mathbf{j} - ye^z\mathbf{k}$, and let S be the surface of the box $\{(x, y, z) \mid 0 \leq x \leq 3, 0 \leq y \leq 2, 0 \leq z \leq 1\}$, oriented outward. Calculate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.