Calculus III Final Exam Review

1. Show that $x^2 + y^2 + z^2 = 2x + 6y$ is the equation of a sphere, and find its center and radius.
2. Find the equation of the plane passing through the points $(1, 0, 4)$, $(5, 1, 5)$, and $(3, 2, 10)$.
3. Consider the curve $\mathbf{r}(t) = \langle t, t^2, \sin(t) \rangle$, $0 \leq t \leq \pi$.
   (a) Find the length of this curve.
   (b) Find the tangent line to the curve at the point $(\pi, \pi^2, 0)$.
4. Find the maximum and minimum values of $f(x, y, z) = x^2y^2z^2$ subject to the constraint $x^2 + y^2 + z^2 = 1$.
5. Find the tangent plane to the surface $x + y + z = e^{xyz}$ at the point $(0, 0, 1)$.
6. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = \langle x + y, y - z, z^2 \rangle$, and C is the curve given by $\mathbf{r}(t) = \langle t^2, t^3, t^2 \rangle$, $0 \leq t \leq 1$.
7. Use Green's Theorem to evaluate $\int_C \cos(y)\, dx + x^2 \sin(y)\, dy$, where C is the rectangle with vertices $(0, 0)$, $(5, 0)$, $(5, 2)$, and $(0, 2)$.
8. Use Stokes' Theorem to evaluate $\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle 2y \cos z, e^x \sin z, xe^y \rangle$, and S is the hemisphere $x^2 + y^2 + z^2 = 9$, $z \geq 0$, oriented upward.
9. Let E be the region bound by the cylinder $y^2 + z^2 = 1$ and the planes $x = -1$ and $x = 2$, and let S be the surface of this region. Use the Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle 3xy^2, xe^z, z^3 \rangle$.