Math 3320 Foundations of Mathematics

Exam 1 Comments

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Definition of Odd

Correct Definition

An integer a is odd if there exists an integer x, such that $a = 2x + 1$.

Incorrect Definition

An integer a is odd if there is a number x, such that $a = 2x + 1$

Incorrect Definition

The definition of odd is an integer that is not divisible by the number two.

Air Force One is the President of the United States.

Definition of Composite

Correct Definition

A positive integer a is composite provided there is an integer b, such that $1 < b < a$, and $b|a$.

Incorrect Definition

An integer is composite if b and a are integers and $1 < b < a$ and $b|a$.

Incorrect Definition

A positive integer a is composite provided there is an integer b, such that $1 < b < a$, and $a|b$.

Incorrect Definition

A positive integer a is composite provided there is an integer b, such that $1 \leq b \leq a$, and $b|a$.

Proof Template for Disproving False Statement

Correct

Find an example where A is true and B is false.

Incorrect

Assume A. Then show that B is false.

Example

Disprove: If x is an integer and $2|x$, then $4|x$.

Problem 9

Disprove: If a, b, and c are integers, and if $a|c$ and $b|c$, then $(a+b)|c$

- Let $a = 3$, $b = 4$, and $c = 12$.
- $a|c$, since $4a = c$.
- $b|c$, since $3a = c$.
- $a + b = 7$, and $7 \not| 12$, since there is no integer x, such that $7x = 12$.
- We have found integers a, b, and c, such that $a|c$ and $b|c$, but $(a+b) \not| c$.
- Therefore, the given statement is false.

The Definition of Divisibility

Definition (Divisible)

- Let a and b be integers.
- We say that a is *divisible* by b provided there is an integer c, such that $bc = a$.
- We also say that b *divides* a, or b is a *factor* of a, or b is a *divisor* of a.
- The notation for this is $b|a$.

Nonsense Statements

- The square root of a triangle is a circle.
- $a|c = b|c$
- $a|c + b|c = z$
- $\frac{a}{b}$ is true.

Problem 8

Show that the Boolean expressions $\neg(x \vee y)$ and $(\neg x) \wedge (\neg y)$ are logically equivalent.

x	y	$x \vee y$	$\neg(x \vee y)$	$\neg x$	$\neg y$	$(\neg x) \wedge (\neg y)$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Since the truth values in columns four and seven are the same, the Boolean expressions $\neg(x \vee y)$ and $(\neg x) \wedge (\neg y)$ are logically equivalent.

Problem 10

Prove the following: Let a, b, and c be integers. If $a|b$ and $a|c$, then $a|(b+c)$.

- Assume that a, b, and c are integers, $a|b$, and $a|c$.
- There exist integers x and y, such that $ax = b$, and $ay = c$, by definition of divisibility.

$$\begin{aligned} a(x+y) &= ax + ay, \text{ by the distributive property (Appendix D)} \\ &= b + c. \end{aligned}$$

- Therefore, $a|(b+c)$, by definition of divisibility.

Incorrect Proof Method: Proof by Example

Prove the following: Let a, b, and c be integers. If $a|b$ and $a|c$, then $a|(b+c)$.

- Let $a = 5$, $b = 10$, and $c = 15$.
- $a|b$, $a|c$, and $a|(b+c)$

- You **can't** prove a general statement with an example.
- You **can** disprove a general statement with a counterexample.

If x is an integer and $2|x$, then $4|x$.

Incorrect Proof Method: "Working Backwards"

Prove the following: Let a, b, and c be integers. If $a|b$ and $a|c$, then $a|(b+c)$.

- Assume that a, b, and c are integers, $a|b$, and $a|c$.
- There exist integers x and y, such that $ax = b$, and $ay = c$, by definition of divisibility.
- Define $z = x + y$

$$az = b + c$$
$$a(x+y) = ax + by$$
$$ax + ay = ax + ay$$

- Since the last statement is true, it proves the first statement is true.

- To prove $A \Rightarrow B$, assume A, and deduce B.
- Don't assume what you are trying to prove.