Foundations of Mathematics
Final Exam Review

1. Be prepared to define the following:
   - $a|b$, where $a, b \in \mathbb{Z}$
   - even
   - odd
   - prime
   - composite
   - List
   - The falling factorial, $(n)_k$
   - The factorial, $n!$
   - Set
   - Cardinality of a set, $|A|$
   - $A \subseteq B$
   - $A = B$, where A and B are both sets
   - Power set
   - $\exists x \in A$, assertions about x
   - $\forall x \in A$, assertions about x
   - $A \cup B$, $A \cap B$, $A - B$, and $A \times B$, for sets A and B
   - Relation
   - Reflexive, Symmetric, Antisymmetric, Transitive
   - Equivalence relation
   - $x \equiv y \pmod{n}$
   - The equivalence class, $[a]$, corresponding to the element $a \in A$.
   - Partition
   - Function
   - Domain, codomain, image (range)
   - one-to-one (injective), onto (surjective), and bijective.
   - countably infinite, uncountably infinite
   - countable
2. Know truth tables for these Boolean operations:
   (a) $x \wedge y$
   (b) $x \vee y$
   (c) $\neg x$
   (d) $x \Rightarrow y$

3. Know the following proof templates
   (a) Direct proof of an if-then statement
   (b) Direct proof of an if-and-only-if statement
   (c) Proving an if-then statement is false with a counterexample
   (d) $A \subseteq B$
   (e) $A = B$, where A and B are both sets
   (f) $\exists x \in A$, assertions about x
   (g) $\forall x \in A$, assertions about x
   (h) Proving a statement is false by proving the negation is true
   (i) Proof by contrapositive.
   (j) Proof by contradiction.
   (k) Proving uniqueness
   (l) The principle of mathematical induction
4. Be prepared to prove/disprove statements.

   Examples:

   (a) Disprove: If $x, y, z \in \mathbb{Z}$, and $x > y$, then $xz > yz$.
   (b) Prove: if $a, b, c \in \mathbb{Z}$, and $a|b$, then $(ac)|(bc)$
   (c) Let A and B be sets, such that $A \subseteq B$. Prove that $2^A \subseteq 2^B$.
   (d) Let R be the relation on $\mathbb{Z}$, such that xRy iff $x|y$. Prove or disprove each of the following:
      i. R is reflexive
      ii. R is symmetric
      iii. R is transitive
      iv. R is antisymmetric
      v. R is an equivalence relation
5. Be prepared to answer questions or perform computations without proof.
   (a) Let $A = \{1, \{3, 4, 9\}, 6\}$, and $B = \{1, 2, 3, \ldots, 99, 100\}$. What is $|2^A \times B|$?
   (b) A salesman is planning to visit eight cities next month from a list of fifty that contains Austin and Dallas. An itinerary for his trip is a list of the eight cities that will be visited in order, without repetitions. How many itineraries start in Austin and end in Dallas?
   (c) Consider the relation $\equiv \pmod 5$, and let [7] be the equivalence class of 7 for this relation. What is $[7] \cap \{20, 21, \ldots, 50\}$?
   (d) Define $f = \{(1,1), (2,3), (3,2), (4,1)\}$.
      i. Is f a function?
      ii. Is f one-to-one?
      iii. If we view f as the function $f : \{1,2,3,4\} \to \{1,2,3\}$, is f onto?
      iv. If we view f as the function $f : \{1,2,3,4\} \to \{1,2,3,4\}$, is f onto?