

Foundations of Mathematics Exam Two Review

1. Be prepared to define the following.

- (a) List
- (b) The falling factorial, $(n)_k$
- (c) The factorial, $n!$
- (d) Set
- (e) Cardinality of a set, $|A|$
- (f) $A \subseteq B$
- (g) $A = B$, where A and B are both sets
- (h) Power set
- (i) $\exists x \in A$, assertions about x
- (j) $\forall x \in A$, assertions about x
- (k) $A \cup B$, $A \cap B$, $A - B$, and $A \times B$ for sets A and B .

2. Applying counting techniques.

- (a) There are n^k ways to select k items from a set of n , if order matters, and repetitions are allowed.
Example: How many 5-letter passwords can be formed using capital and lowercase letters and digits?
- (b) There are $(n)_k$ ways to select k items from a set of n , if order matters, and repetitions are not allowed.
Example: How many ways are there to assign 9 baseball players to defensive positions, if 20 are available?
- (c) $|A \cup B| = |A| + |B| - |A \cap B|$
Example: How many 5-letter passwords can be formed using capital and lowercase letters and digits, where the first character is "T" or the last character is "7"?

3. Know the following proof templates.

- (a) $A \subseteq B$
- (b) $A = B$, where A and B are both sets
- (c) $\exists x \in A$, assertions about x
- (d) $\forall x \in A$, assertions about x
- (e) Proving a statement is false.

4. Be prepared to write proofs using those templates.

Examples:

- (a) Let A , B , and C be sets, and assume that $A \subseteq B$. Prove that $A \times C \subseteq B \times C$.
- (b) Prove that $\exists x \in \mathbb{Z}, x^2 + x + 19$ is composite.
- (c) Let A be the set of all odd integers. Prove that $\forall x \in A, x^2$ is odd.
- (d) Disprove: $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, xy = 1$.