Foundations of Mathematics Exam Three Review

1. Be prepared to define the following:

   (a) Relation

   (b) Reflexive, Symmetric, Antisymmetric, Transitive

   (c) Equivalence relation

   (d) $x \equiv y \pmod{n}$

   (e) The equivalence class, $[a]$, corresponding to the element $a \in A$.

   (f) Partition

   (g) Function

   (h) Domain, codomain, image (range)

   (i) one-to-one (injective), onto (surjective), and bijective.

   (j) countably infinite, uncountably infinite

   (k) countable

2. Know the following proof templates

   (a) Proof by contrapositive.

   (b) Proof by contradiction.

   (c) Proving uniqueness

   (d) The principle of mathematical induction

3. Be prepared to write proofs.

   Examples:

   (a) Prove that congruency mod n is a symmetric relation.

   (b) Prove that if $[a] \cap [b] \neq \emptyset$, then $[a] = [b]$.

   (c) Prove that the function

   $$f : \mathbb{R} \to \mathbb{R}$$
   $$f(x) = 3x + 7$$

   is a bijection.

   (d) Prove that if n is a positive integer, then $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

4. For each of the following sets, state whether it is countable or uncountable (you don't have to prove it).

   (a) $\{1, 2, 3\}$

(b) $\mathbb{N}$

(c) $\mathbb{Z}$

(d) $\mathbb{Q}$

(e) $\mathbb{R}$