

## Probability and Statistics Exam Three Review

Name: \_\_\_\_\_

Please show your work on all problems.

1. **60 pts.** The probability density function of  $X$  is  $f(x) = cx^3$ , for  $0 \leq x \leq 5$ .

(a) Find  $c$ .

(b) Find the cumulative distribution function of  $X$ .

(c) Calculate  $P(1 \leq X \leq 4)$

(d) Find the expected value of  $X$

(e) Find the variance and standard deviation of  $X$ .

(f) Express the moment-generating function of  $X$  as an integral (you don't have to evaluate the integral).

2. **50 pts.** A radioactive sample emits alpha particles according to a Poisson process at a rate of  $\frac{1}{8}$  particles per second.

(a) Let  $W$  be the waiting time in seconds until the first particle is emitted. What type of distribution does  $W$  have? What is the probability density function for  $W$ ?

(b) What are the expected value and variance of  $W$ ?

(c) What is the probability of waiting between 6 and 11 seconds for the first particle to be emitted ?

(d) Suppose  $W_5$  is the waiting time until the fifth particle is emitted. What type of distribution does  $W_5$  have? Find the probability density function of  $W_5$ .

(e) Find the expected value and variance of  $W_5$ .

(f) What is the probability of waiting between 30 and 50 seconds for the fifth particle to be emitted ?

3. **20 pts.** A certain population of people have heights that are normally distributed with a mean of 70 inches and a standard deviation of 2.5 inches.

(a) What percentage of these people have heights between 66 and 72 inches?

(b) Find the 80th percentile of heights in this population.

4. Let  $Z$  be a standard normal variable.

(a) Compute  $P(-1 \leq Z \leq 1.7)$ .

(b) Compute  $z_{0.005}$ .