Calculus III Review One

1. Show that $x^2 + 8z + 16y + z^2 = -y^2 + 10x$ is the equation of a sphere, and find its center and radius.

2. Let $\mathbf{a} = \langle 2, 4, -5 \rangle$, $\mathbf{b} = \langle 6, -3, 2 \rangle$, and $\mathbf{c} = \langle 0, 0, 4 \rangle$. Determine the following:

   (a) $5\mathbf{a} - 3\mathbf{b}$.

   (b) The vector projection of $\mathbf{b}$ onto $\mathbf{a}$.

   (c) Are $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ coplanar?

3. Find the equation of the plane passing through the points $P(1, 8, 2)$, $Q(5, 0, 1)$, and $R(3, 5, -1)$, and find the area of the triangle PQR.

4. (a) Find the parametric and symmetric equations of the line through $A(1, 0, 1)$ and $B(4, -2, 2)$.

   (b) Where does this line intersect the plane $x + y + z = 6$?

   (c) What is the angle between this line and plane?

5. Sketch and describe the graph of $z^2 - x^2 = 1$ as a surface in $\mathbb{R}^3$.

6. Sketch and describe the graph of $y^2 = x^2 + 2z^2$ as a surface in $\mathbb{R}^3$.