Probability and Statistics II Review 2

1. Let X be a random variable with probability density $f(x) = \frac{1}{39}x^2$, for $2 \leq x \leq 5$. Find the probability density of $Y = \ln(X)$.

2. Let $X_1, \ldots, X_k$ be statistically independent random variables, and assume that X_i has a binomial distribution with parameters n_i and p, for each $i = 1, \ldots, k$. What is the probability distribution of $Y = X_1 + \cdots + X_k$?

3. Assume that scores on the Math SAT are approximately normally distributed with mean 500 and standard deviation 100, and consider a sample of 20 students.

   (a) Find $P(470 \leq \overline{X} \leq 530)$.

   (b) Find constants a and b, such that $P(a \leq S \leq b) = 0.95$.

   (c) For the z-statistic defined below, find $P(-2 \leq Z \leq 2)$.

   $$Z = \frac{\overline{X} - 500}{100/\sqrt{20}}.$$

   (d) For the t-statistic defined below, find $P(-2 \leq T \leq 2)$.

   $$T = \frac{\overline{X} - 500}{S/\sqrt{20}}.$$

4. A portfolio manager invests \$100 in each of 300 statistically independent stocks. Let $X_1, \ldots, X_{300}$ denote the future value of these stocks in dollars after ten years, and assume that $X_i \sim U(0, 400)$, for $i = 1, \ldots, 300$.

   (a) Find the mean and standard deviation of the future value of one stock.

   (b) What is the probability that an individual stock increases by at least 85%, i.e., what is $P(X_i \geq 185)$?

   (c) Find the mean and standard deviation of the future value of the entire portfolio, $Y = X_1 + \cdots + X_{300}$.

   (d) What is the probability that the entire portfolio increases by at least 85%, i.e., what is $P(Y \geq 55,500)$?