Moment Generating Function Technique

Solve the problems below using the moment-generating-function technique. Make sure to state the distribution and its parameters.

1. Let $X_1, \ldots, X_n$ be independent random variables, such that $X_i \sim \text{Exponential}(\theta)$, for $i = 1, \ldots, n$. Find the distribution of

$$Y = X_1 + \cdots + X_n.$$

2. Let $X_1, \ldots, X_n$ be independent random variables, such that $X_i \sim \text{Poiss}(\lambda_i)$, for $i = 1, \ldots, n$. Find the distribution of

$$Y = X_1 + \cdots + X_n.$$

3. Let $X_1, \ldots, X_n$ be independent random variables, such that $X_i \sim N(\mu_i, \sigma_i^2)$, for $i = 1, \ldots, n$. Find the distribution of

$$Y = a_1 X_1 + \cdots + a_n X_n.$$