**Calculus I Lab Activity: Precise Definition of a Limit**

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1. **Go to the WebAssign page:** [**http://www.webassign.net/**](http://www.webassign.net/) **and log in**.
2. **Click on the “My eBooks” item on the horizontal toolbar near the top of the page**. A popup window should appear. **Click on the “Calculus: Early Transcendentals – 7e Stewart” link**.
3. **Click on the “Media” button on the horizontal toolbar near the top of the page**. There’s typically a bit of a delay at this point while the “Media” menu is loaded.
4. **Under the “Media” tab in the left frame, click on “Chapter 2 Media”**. An extensive list of media items available to you are presented, including Lecture Videos, TEC (Tools for Enriching Calculus), Wolfram Demonstrations, Video Examples, and Homework Helpers. These are worth your exploration when you have some time (later).
5. **Under the TEC grouping, choose the “2.6 M2.4/2.6 Precise Definitions of Limits (p.139)”**. At this point, the frame on the right should take you to a snapshot of p. 139 in the textbook, which has a pink “TEC” button in the upper left corner. **Click on the TEC button on the top left corner of p. 139.** A new window should appear. **Maximize this window** and let’s get to work!

Example:

* Click “The Limit of a Function” and select the first function, $=x^{3}-5x+6$ .
* Uncheck the “Grid” box.
* Use the horizontal slider below the graph the set $a=2$. Notice that the value of $a$ is shown to the right of that slider bar. Make sure that it is precisely shown to be 2.00.
* The graph suggests that $\lim\_{x\to 2}x^{3}-5x+6=4$.
* The precise definition of the limit requires that:
	+ for **every positive number** $ϵ, $
	+ **there must exist a positive number** $δ$ **such that if** $x$ **is within** $δ$ **of 2** (i.e., if $2-δ<x<2+δ$ or, equivalently, $\left|x-2\right|<δ$),
	+ **then** $f(x)$ **is within** $ϵ$ **of 4** (i.e., $4-ϵ<f(x)<4+ϵ$ or, equivalently, $\left|f\left(x\right)-4\right|<ϵ$ ).
* Let’s explore this relationship by first using the vertical slider to set $ϵ=1$. Again, the value of $ϵ$ is shown above the vertical slider. Graphically, notice that as you dynamically modify the value of $ϵ$ with the slider, the two horizontal lines $y=4+ϵ$ and $y=4-ϵ$ are changing accordingly. NOTE: The points on the graph of the function that lie between these two horizontal lines represent the ordered pairs in the function where the output values ($f(x))$ are between $4+ϵ$ and $4-ϵ$.
* Then use the smaller horizontal slider bar below the graph to adjust the value of $δ$ so that it bounds the input (or $x$) values only to those near 2 for which the corresponding output ($f(x)$) values are within $ϵ$ of 4 (i.e., so that the points on the graph of the function near $x=2$ are between the two horizontal lines). **We typically choose** $δ$ **to be the largest number that satisfies this condition**. So, for $ϵ=1$, what is $δ$?
* Now, click on the “Zoom In” button. For $ϵ=0.1$, what is $δ$?

In the time remaining, repeat this exercise for different choices of function, $a$, $ϵ$, and $δ$ to respond to the final questions.

Final Questions:

* + In general, how does $δ$ change as $ϵ$ gets smaller?
	+ How does this relationship between $ϵ$ and $δ$ change if the related limit does not exist?