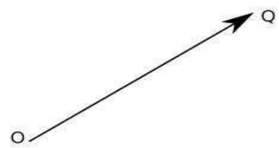
# Vectors: Forms, Notation, and Formulas

A **scalar** is a mathematical quantity with magnitude only (in physics, mass, pressure or speed are good examples). A **vector quantity** has magnitude **and** direction. Displacement, velocity, momentum, force, and acceleration are all vector quantities. Two-dimensional vectors can be represented in three ways.

## Geometric

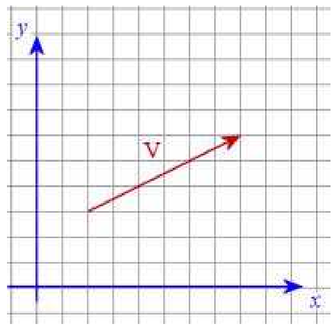
Here we use an arrow to represent a vector. Its length is its **magnitude**, and its direction is indicated by the direction of the arrow.




The vector here can be written **OQ** (bold print) or $\overrightarrow{OQ}$ with an arrow above it. Its **magnitude** (or length) is written $|\mathbf{OQ}|$ (absolute value symbols).

## Rectangular Notation $\langle a, b\rangle$

A vector may be located in a rectangular coordinate system, as is illustrated here.




The rectangular coordinate notation for this vector is $\mathbf{v} = \langle 6, 3\rangle$ or $\vec{v} = \langle 6, 3\rangle$. Note the use of **angle brackets** here.

An alternate notation is the use of two **unit vectors** $\hat{i} = \langle 1, 0\rangle$ and $\hat{j} = \langle 0, 1\rangle$ so that

$$\mathbf{v} = 6\hat{i} + 3\hat{j}$$

The "hat" notation, not used in our text, is to indicate a unit vector, a vector whose magnitude (length) is 1.

## Polar Notation $\langle r\angle\theta\rangle$

In this notation we specify a vector's magnitude $r$, $r \geq 0$, and its angle $\theta$ with the positive $x$-axis, $0° \leq \theta < 360°$. In the illustration above, $r \approx 6.7$ and $\theta \approx 27°$ so that we can write

$$\vec{v} = \langle 6.7\angle 27°\rangle$$

## Conversions Between Forms

### Rectangular to Polar

If $\mathbf{v} = \langle a, b\rangle$ then

$$|\mathbf{v}| = \sqrt{a^2 + b^2} \quad \text{and}$$

$$\tan\theta = \frac{b}{a},\ a \neq 0, \quad \text{and } (a, b) \text{ locates the quadrant of } \theta$$

If $a = 0$ and $b > 0$, then $\theta = 90°$. If $a = 0$ and $b < 0$, then $\theta = 270°$.

**Polar to Rectangular**

If $\mathbf{v} = \langle r\angle\theta \rangle$ then

$$\mathbf{v} = \langle r\cos\theta, r\sin\theta \rangle$$

## Vector Operations

**Scalar Multiplication**

Geometrically, a scalar multiplier $k > 0$ can change the length of the vector but not its direction. If $k < 0$, then the scalar product will "reverse" the direction by $180°$.

In rectangular form, if $k$ is a scalar then

$$k\langle a, b \rangle = \langle ka, kb \rangle$$

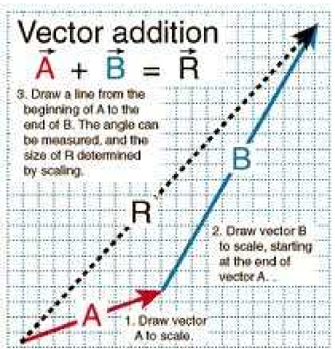
In the case of a polar form vector

$$k\langle r\angle\theta \rangle = \begin{cases} \langle kr\angle\theta \rangle & if \quad k \geq 0 \\ \langle |kr|\angle\theta \pm 180° \rangle & if \quad k < 0 \end{cases}$$

In the case where $k < 0$, choose $\theta + 180°$ if $0° \leq \theta < 180°$. Choose $\theta - 180°$ if $180° \leq \theta < 360°$

**Vector Addition**

In geometric form, vectors are added by the **tip-to-tail** or **parallelogram** method.




In rectangular form, if $\mathbf{u} = \langle a, b \rangle$ and $\mathbf{v} = \langle c, d \rangle$ then

$$\mathbf{u} + \mathbf{v} = \langle a + c, b + d \rangle$$

It's easy in rectangular coordinates. The sum of two vectors is called the **resultant**.

In polar coordinates there are two approaches, depending on the information given.

1. Convert polar form vectors to rectangular coordinates, add, and then convert back to polar coordinates.
2. If the magnitudes of the two vectors and the angle between is given (but not the directions of each vector), then a triangle sketch with a Law of Cosines solution is used.

**Vector Dot Product**

If $\mathbf{u} = \langle a, b \rangle$ and $\mathbf{v} = \langle c, d \rangle$ then the **dot product** of **u** and **v** is

$$\mathbf{u} \cdot \mathbf{v} = ac + bd$$

The dot product may be positive real number, 0, or a negative real number.

If the magnitudes of the two vectors are known and the angle $\theta$ between them is known, then

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos\theta$$

This last formula can be used to find the angle between two vectors whose rectangular forms are given

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$