Properties of Logarithm Functions

These properties follow from the definition of a logarithm function as the inverse of an exponential function, and from the exponent rules.

1. $\log_b 1 = 0$ [zero power rule: $b^0 = 1$]

2. $\log_b b = 1$ [$b^1 = b$]

3. $\log_b b^x = x$ [composition of a function with its inverse is the identity function]

4. $b^{\log_b x} = x$ [composition of a function with its inverse is the identity function]

5. $\log_b M = \log_b N$ if and only if $M = N$ [log functions are one-to-one].

6. $\log_b(M \cdot N) = \log_b M + \log_b N$ [from the addition of exponents rule: $a^m a^n = a^{m+n}$]

7. $\log_b\left(\dfrac{M}{N}\right) = \log_b M - \log_b N$ [from the subtraction of exponents rule: $\dfrac{a^m}{a^n} = a^{m-n}$]

8. $\log_b M^p = p\log_b M$ [from the power rule: $(a^m)^n = a^{mn}$]