Notes: **Using the Half-Angle Formulas**

Refer to the Formula List for these examples.

Example: If $\cos(A) = -0.91$ and $\sin(A) < 0$ determine the following:

1. The quadrant for angle $A/2$ = ?.

Because cosine and sine are both negative, A must be in QIII.

$$180° < A < 270°$$
$$90° < A/2 < 135°$$

So $A/2$ is in QII.

2. $\sin(A)$ = ?

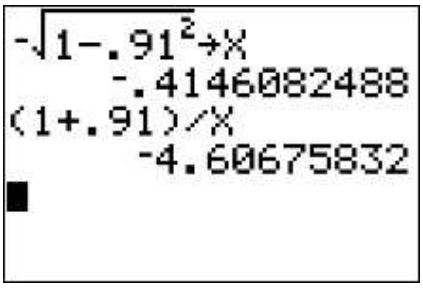
Since A is in QIII, $\sin(A)$ is negative and

$$\begin{aligned}\sin(A) &= -\sqrt{1-\cos^2(A)} \\ &= -\sqrt{1-(-0.91)^2} \\ &= -0.41461\end{aligned}$$

It's a good idea to store this answer on your calculator because you will need it to find $\tan(A/2)$. Let's go ahead and do that part now. Remember that **X** is a good temporary location as long as you don't do a graph.

$$\begin{aligned}\tan\frac{A}{2} &= \frac{1-\cos A}{\sin A} \\ &= \frac{1-(-0.91)}{-0.41461} \\ &= -4.6068\end{aligned}$$

Here's the calculator view. Notice that there are some sign simplifications before using the calculator.



3. $\sin(A/2)$ = ?

$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$ Since $A/2$ is in QII we choose +.

$$\begin{aligned}\sin(A/2) &= +\sqrt{\frac{1-(-0.91)}{2}} \\ &= 0.97724\end{aligned}$$

4. $\cos(A/2) = ?$

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$ Since $A/2$ is in QII we choose $-$.

$$\begin{aligned}\cos\frac{A}{2} &= -\sqrt{\frac{1+\cos A}{2}}\\ &= -\sqrt{\frac{1+(-0.91)}{2}}\\ &= -0.212\,134\end{aligned}$$

Example: Given $\sin(a) = -7/8$ and a is in quadrant IV, find the **exact** value of $\sin(a/2)$.
Note: You are not allowed to use decimals in your answer.

To use the half-angle formulas we must find $\cos(a)$. Since a is in QIV, cosine is positive and

$$\begin{aligned}\cos(a) &= +\sqrt{1-\left(\frac{7}{8}\right)^2}\\ &= \sqrt{\frac{15}{64}}\end{aligned}$$

Since angle a is in QIV we have

$$\begin{aligned}270^\circ &< a < 360^\circ\\ 135^\circ &< a/2 < 180^\circ\end{aligned}$$

and so $a/2$ is in QII and $\sin(a/20)$ is positive.

$$\begin{aligned}\sin\frac{a}{2} &= +\sqrt{\frac{1-\cos a}{2}}\\ &= \sqrt{\frac{1-\sqrt{\frac{15}{64}}}{2}}\end{aligned}$$

Here's how to type this for an online answer. Note carefully how the ( ) are paired up.

sqrt((1-sqrt(15/64))/2)

Example: Given $tan(a) = \frac{8}{\sqrt{17}}$ and a is in quadrant III, find the exact value of $tan(a/2)$.
Note: You are not allowed to use decimals in your answer.

Since a is in QIII both x and y are negative.

$$\begin{aligned}x &= -\sqrt{17}\\ y &= -8\\ r &= \sqrt{\left(-\sqrt{17}\right)^2+(-8)^2}\\ &= \sqrt{17+64}\\ &= \sqrt{81}\\ &= 9\end{aligned}$$

Using this, we have

$$\sin(a) = -\frac{8}{9}$$

$$\cos(a) = -\frac{\sqrt{17}}{9}$$

and

$$\begin{aligned}\tan\frac{a}{2} &= \frac{1-\cos a}{\sin a}\\ &= \frac{1-\left(-\frac{\sqrt{17}}{9}\right)}{-\frac{8}{9}}\\ &= \frac{9+\sqrt{17}}{-8}\\ &= -\frac{9+\sqrt{17}}{8}\end{aligned}$$

The online answer form is

-(9+sqrt(17))/8