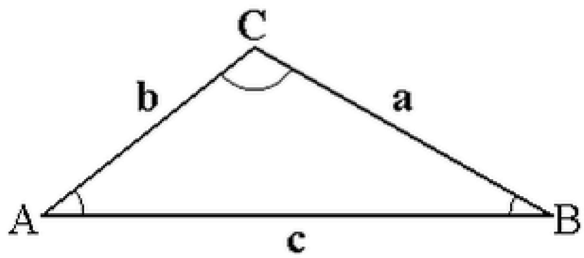
Notes: Solving an Oblique Triangle with the Law of Sines

An **oblique triangle** is a triangle that does not have a right angle. To deal with this situation we have The Law of Sines and the Law of Cosines. In these notes we will use the Law of Sines to handle two types of problems.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

ASA (angle-side-angle) given

Example: Consider the triangle below.



If $c = 8$, the angle $C = 120°$ and the angle $B = 35°$ find the other angle A and the remaining sides a and b. Give your answer to at least 3 decimal places.

Solution: We first find the third angle A

$$A = 180° - 120° - 35°$$
$$A = 25°$$

Then we can use these three angles and the Law of Sines to find the unknown sides a and b.

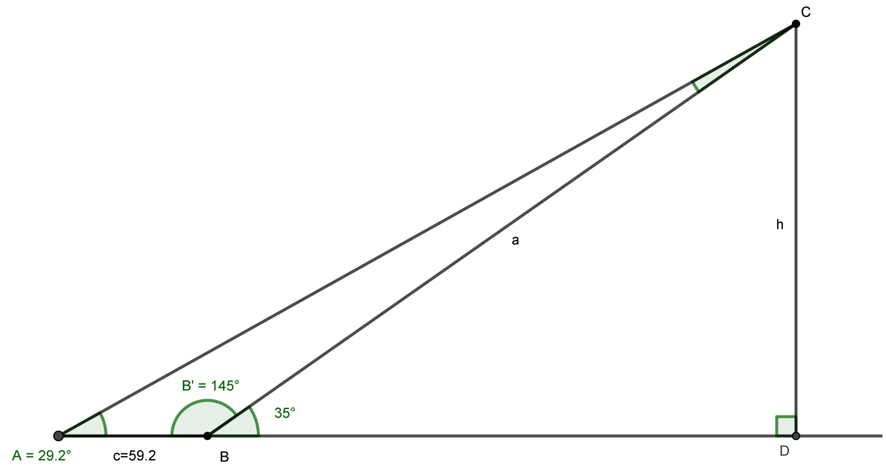
$$\frac{8}{\sin 120°} = \frac{a}{\sin 25°}$$
$$a = \frac{8 \sin 25°}{\sin 120°}$$
$$a = 3.9040$$

To set up the solution for side b, be sure to use an equation with original given information. If possible, avoid using a solution for one side to find another side.

$$\frac{8}{\sin 120°} = \frac{b}{\sin 35°}$$
$$b = \frac{8 \sin 35°}{\sin 120°}$$
$$b = 5.2985$$

Example: A surveyor determines that the angle of elevation to the top of a building from a point on the ground is $35°$. He then moves back 59.2 feet and determines that the angle of elevation is $29.2°$. What is the height of the building? Round your answer to four decimal places.

Solution: Here's a sketch of the situation, approximately to scale.



We consider $\triangle AB'C$ where $\angle A = 29.2°$ and

$$\angle B' = 180° - 35° = 145°$$

$$\angle C = 180° - 29.2° - 145° = 5.8°$$

We want find the length of side $a = BC$ which is opposite $\angle A$. We will use the Law of Sines

$$\frac{59.2}{\sin 5.8°} = \frac{a}{\sin 29.2°}$$

$$a = \frac{59.2 \sin 29.2°}{\sin 5.8°}$$

$$a = 285.7941$$

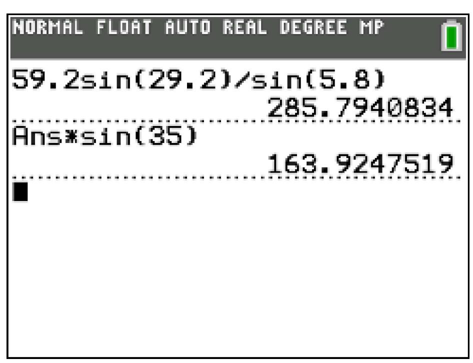
We can now find side h in the right triangle BCD by seeing side a as the hypotenuse. Then

$$\sin 35° = \frac{h}{a} = \frac{h}{285.7941}$$

$$h = 285.7941 \sin 35°$$

$$h = 163.9248$$

In these steps we have used one answer for side a to find a second answer for h. In the calculator,do these steps back-to-back with the **ANS** variable to insure that your answer is as accurate as possible.



To bring up the **ANS** variable after the first line just type the × symbol on the keyboard.

The next example involve the SSA (side-side-and opposite angle given) case. There are six possible situations which will be considered in the examples in the next note set.