Notes on Periodic Functions

We have previously discussed even and odd functions. For the trig functions, $\sin(x)$ and $\tan(x)$ are **odd** functions:

$$\sin(-x) = -\sin(x)$$

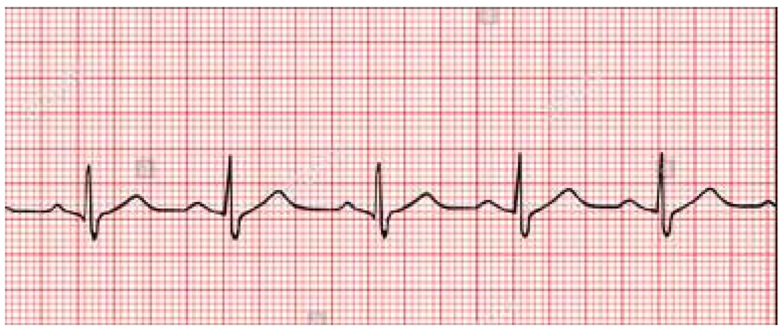
$$\tan(-x) = -\tan(x)$$

and $\cos(x)$ is even:

$$\cos(-x) = \cos(x)$$

Functions in general may be odd, even, or neither.

Another property of functions that we see in trigonometry is that of being **periodic**. We see this property in the graph of an EKG:



Definition: A function $f(x)$ is periodic if there is a positive number p such that

$$f(x+p) = f(x)$$

for all x in the domain of f. If there is a *smallest* value p for which this holds, then it is called **the period of** f.

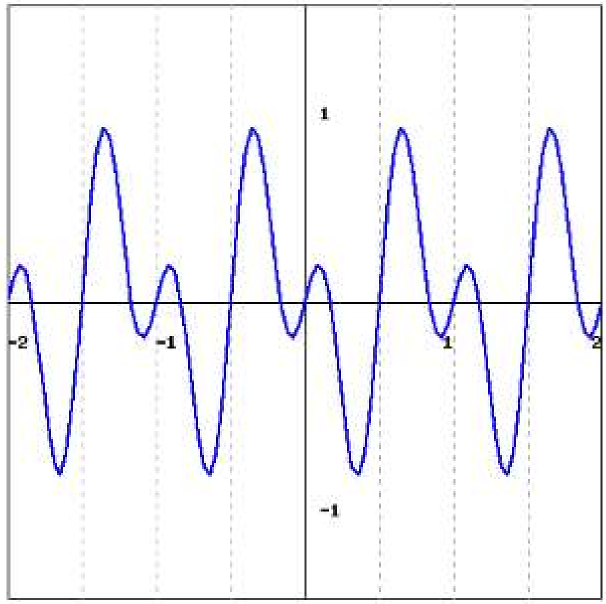
Examples: The functions $\sin(x)$ and $\cos(x)$ are periodic with period 2π (or 360 degrees in degree mode). The tangent function $\tan(x)$ is also periodic. Its period π (or 180 degrees)! Graph these with your calculator or Desmos to see the period. Use **radian mode** on your calculator and **Zoom 7. Trig** .

Example: Could this table represent a periodic function?

x	0	4	8	12	16	20	24	28
$g(x)$	0.9	7.4	-1.5	0.9	7.4	-1.5	0.9	7.4

Tracking values that repeat, we see that 0.9 is $g(0)$, $g(12)$, and $g(24)$. This suggests that the period is 12 and if we check the next output number, 7.4, we see that is also repeats after after 12 input steps, as does -1.5. So it could be a table from a periodic function.

Example: Is this the graph of a periodic function?



Yes, the graph shows a repeating pattern. Examining the *x*-axis scale shows that it repeats every 1 unit. Also, the distance between successive "peaks" or "valleys" is one unit. The period is 1.