Quadratic Functions and Their Graphs

Definition A *quadratic function* is a function that can be written in the form

$$f(x) = ax^2 + bx + c, \ a \neq 0$$

This form is called the **Standard Form**. The number a is called the **leading coefficient**.

General Notes on the graphs of quadratic functions:

1. The graph is a *parabola* which opens upward if $a > 0$ and opens downward if $a < 0$. The absolute value of a determines the shape of the parabola. If $|a| > 1$, then the graph will get "thinner" as a becomes larger. If $|a| < 1$, then the graph becomes "wider" as a approaches 0.
2. The y-intercept of the graph is $y = c$. The y-intercept point is $(0, c)$.
3. The x-intercepts are found by solving the quadratic equation

$$ax^2 + bx + c = 0$$

   for x. Each *real* solution gives an x-intercept.
4. The most important point on the graph is the **vertex**, typically labeled (h, k). The vertex can be found by
   - **A.** Completing the square method, or
   - **B.** Using the formula

$$h = -\frac{b}{2a}$$

   This gives the x-coordinate of the vertex, which we have labeled h. To find the y-coordinate of the vertex (labeled k), substitute this value for x in the quadratic function and simplify. This value will be a **maximum** or **minimum** for y.
5. The *axis of symmetry* of the graph is the vertical line

$$x = -\frac{b}{2a}$$

   which passes through the vertex.

Factored Form of a Quadratic Function

If r and s are real numbers, with $r \leq s$, then the **factored form** of a quadratic function is

$$f(x) = a(x - r)(x - s)$$

The orientation and shape of the graph of this function are determined by the leading coefficient a. The two x-intercepts of the graph are $(r, 0)$ and $(s, 0)$. We typically will just say "x-intercepts are r and s."

Vertex Form of a Quadratic Function

If the vertex (h, k) is known, the quadratic function can be written in **Vertex Form**

$$f(x) = a(x - h)^2 + k$$

Conversely, the vertex, orientation, and shape of the parabola can be read from this form.

1. If $a > 0$, then the graph of the function opens upward and the number k is the **minimum value** of the range of f.
2. If $a < 0$, then the graph of the function opens downward and the number k is the **maximum value** of the range of f.

The standard form inherently carries the least information. We usually want to convert from this form to the vertex form or the factored form.