Notes and Examples: Using the Sum and Difference Formulas for Sine and Cosine

It is easy to think that the following is an identity:

$$\sin(A + B) = \sin(A) + \sin(B)$$

But if we let $A = 30°$ and $B = 60°$ then

$$\sin(30° + 60°) = \sin(30°) + \sin(60°)$$

$$\sin(90°) = \frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$1 = \frac{1 + \sqrt{3}}{2} \quad \text{False!}$$

For the correct formulas, refer to the **Formula List** document for these exercises.

Example 1: Use an identity to find the exact value of each expression:
Note: You are not allowed to use decimals in your answer.

$\sin(13°)\cos(17°) + \cos(13°)\sin(17°) = ?$

$\sin(269°)\cos(29°) - \cos(269°)\sin(29°) = ?$

Solution: The first formula represents $\sin(13° + 17°) = \sin(30°) = \frac{1}{2}$

The second formula represents $\sin(269° - 29°) = \sin(240°) = -\frac{\sqrt{3}}{2}$

Example 2: Use a sum or difference identity to find the exact value of each expression:
Note: You are not allowed to use decimals in your answer.

$\sin(195°) = ?$

Solution: We must write $195°$ in terms of angles on the unit circle for which we have exact values:

$$195° = 150° + 45°$$

Other choices, such as $135° + 60°$ can also be made. Then

$$\begin{aligned}
\sin(195°) &= \sin(150° + 45°) \\
&= \sin(150°)\cos(45°) + \cos(150°)\sin(45°) \\
&= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\
&= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} \quad \text{(multiply radicals)} \\
&= \frac{\sqrt{2} - \sqrt{6}}{4}
\end{aligned}$$

For an online answer in this example, you can type:

(sqrt(2)-sqrt(6))/4

and be sure to use parentheses correctly.

Example 3: The function $\sin\left(\frac{\pi}{2} - x\right)$ can be written (simplified) as ?

Solution: This problem uses radian mode.

$$\begin{aligned}\sin\left(\frac{\pi}{2} - x\right) &= \sin\left(\frac{\pi}{2}\right)\cos(x) - \cos\left(\frac{\pi}{2}\right)\sin(x)\\ &= (1)\cos(x) - (0)\sin(x)\\ &= \cos(x)\end{aligned}$$

Another way to check this is to look at the graph of $y = \sin\left(\frac{\pi}{2} - x\right)$ and see that it's the same as the graph of $y = \cos(x)$.

Example 4: Given $\sin(\alpha) = -\frac{2}{9}$ and α is in quadrant IV and $\cos(\beta) = \frac{\sqrt{9}}{5} = \frac{3}{5}$ (we get an exact radical here!)

and β is in quadrant IV. Use sum and difference formulas to find the following:
Note: You are not allowed to use decimals in your answer.

$\sin(\alpha + \beta) = ?$

Solution: We must find the sine and cosine values for angle α and for angle β. First find $\cos(\alpha)$. Since

$$\sin(\alpha) = \frac{y}{r} = -\frac{2}{9}$$

and α is in QIV, x must be positive and y must be negative (remember $r > 0$, always).

$$x = ?$$
$$y = -2$$
$$r = 9$$

Since

$$x^2 + (-2)^2 = 9^2$$

we get

$$x = \sqrt{77} \text{ and}$$
$$\cos(\alpha) = \frac{\sqrt{77}}{9}$$

Next we must find $\sin(\beta)$. Since β is in QIV, x is positive and y is negative

$$x = \sqrt{9} = 3 \text{ an exact radical in this problem}$$
$$y = ?$$
$$r = 5$$

Since

$$3^2 + y^2 = 5^2$$

we get

$$y = -4$$
$$\sin(\beta) = -\frac{4}{5} \text{ remember ASTC}$$

We now assemble all the function values we need to find $\sin(\alpha + \beta)$:

$$\begin{aligned}
\sin(\alpha+\beta) &= \sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta) \\
&= \left(-\frac{2}{9}\right)\left(\frac{3}{5}\right)+\left(\frac{\sqrt{77}}{9}\right)\left(-\frac{4}{5}\right) \\
&= -\frac{6}{45}+\left(-\frac{4\sqrt{77}}{45}\right) \\
&= \frac{-6-4\sqrt{77}}{45}
\end{aligned}$$

For an online answer type

(-6-4sqrt(77))/45