Examples of Sinusoid Graphs–Transformations of Sine and Cosine

We will use the information we have developed to do two kinds of problems:

1. Given a function of the form $y = A\sin(B(x-h)) + k$ or $y = A\cos(B(x-h)) + k$ find the key features of the graph.
2. Given a graph of a sinusoid, find a formula for it in terms of sine or cosine.

Example: Find the amplitude, period, and phase shift for the graph of

$$y = -17\cos(8\pi x + 9)$$

Solution: The amplitude is $A = |-17| = 17$, a positive number. The period is $\frac{2\pi}{8\pi} = \frac{1}{4}$. To find the phase shift, rewrite the function

$$y = -17\cos(8\pi x + 9) = -17\cos\left(8\pi\left(x - \frac{-9}{8\pi}\right)\right)$$

This shows that the phase shift is $-\frac{9}{8\pi}$, a shift to left.

Example: Find the amplitude, period, phase shift, and midline of the function

$$y = -8\cos(6x) - 3$$

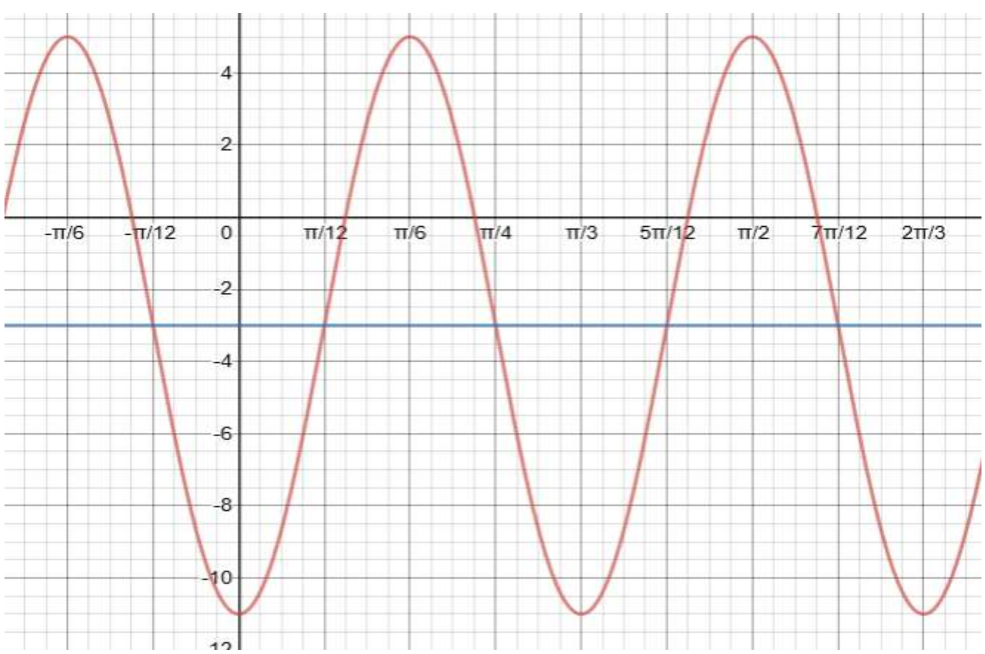
Solution: The amplitude is $|-8| = 8$. The period is $\frac{2\pi}{6} = \frac{\pi}{3}$. Rewriting the function as

$$y = -8\cos(6(x-0)) - 3$$

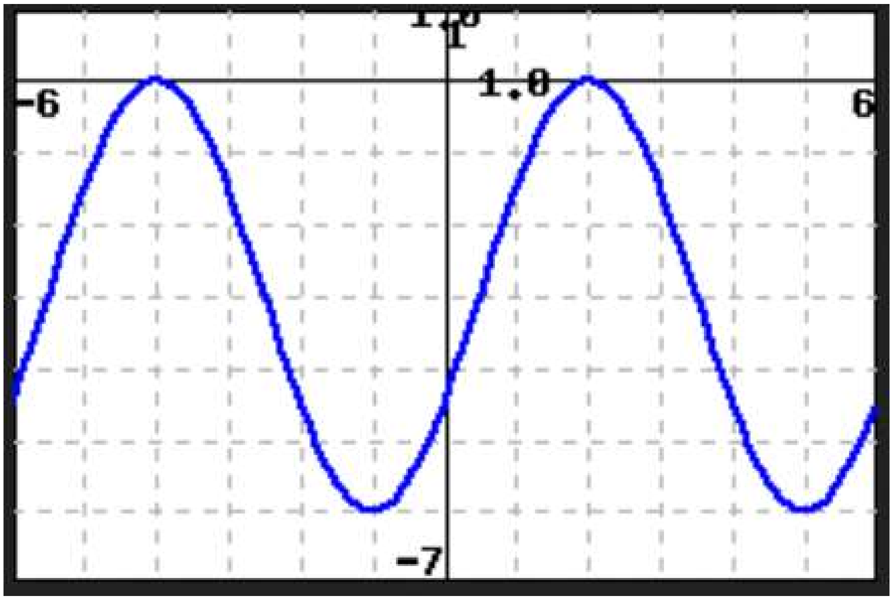
shows that the phase shift is 0 (no shift). The midline is

$$y = -3$$

The graph is a cosine function that has been stretched vertically by a factor of 8, reflected about the x-axis, squeezed horizontally by a factor $\frac{1}{6}$, and shifted down 3 units. Since the period is $P = \frac{\pi}{3}$ the critical value occur at multiples of $\frac{P}{4} = \frac{\pi}{12}$. Here's a Desmos graph with custom scaling to show these features. Note the reflected cosine pattern of the critical points: low, med, high, med, low.



Example: Find a formula for this graph of a trigonometric function.



The function may be written as $f(x) = a\sin\left(\frac{2\pi}{P}(x-b)\right) + c$ or $f(x) = a\cos\left(\frac{2\pi}{P}(x-b)\right) + c$. Hint: a, b, c, and P can be chosen to be integers.

Solution: The high value is 0 and the low value is –6. Therefore the amplitude

$$a = 3$$

and the midline

$$y = c = -3$$

Now look for a convenient (integer) point on the midline. This would let us use the sine function. But none are apparent and we must use the cosine.

Therefore look for a convenient high value and we see that at $x = 2, y = 0$, that is $(2, 0)$ is a high point on the graph. Since the cosine graph starts at the high value, it must be that we have a cosine graph shifted to the right 2 units, so

$$b = 2$$

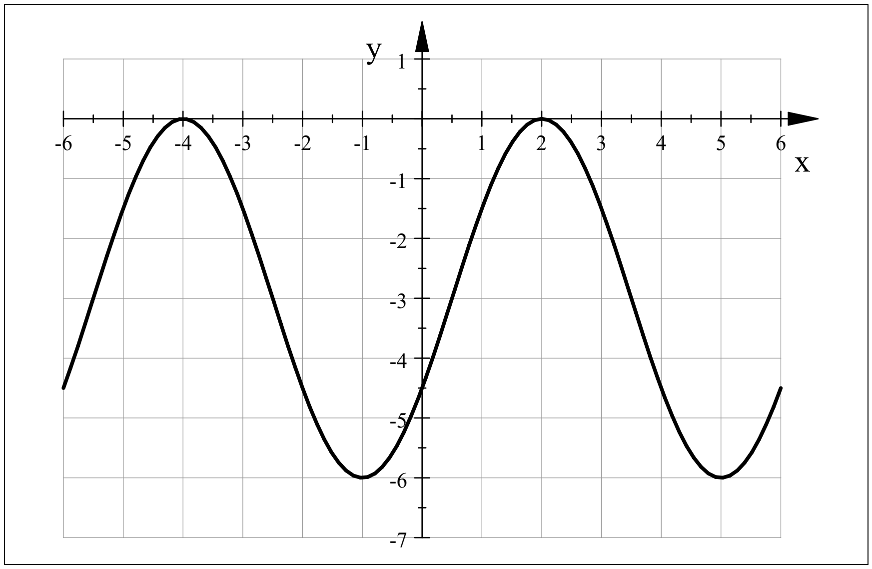
Find the period P by looking at the distance between successive peaks from $x = -4$ to $x = 2$. The period

$$P = 6$$

The information we need is complete and we can write

$$f(x) = 3\cos\left(\frac{2\pi}{6}(x-2)\right) - 3$$

Here's a graph of $f(x)$ which matches the given graph.



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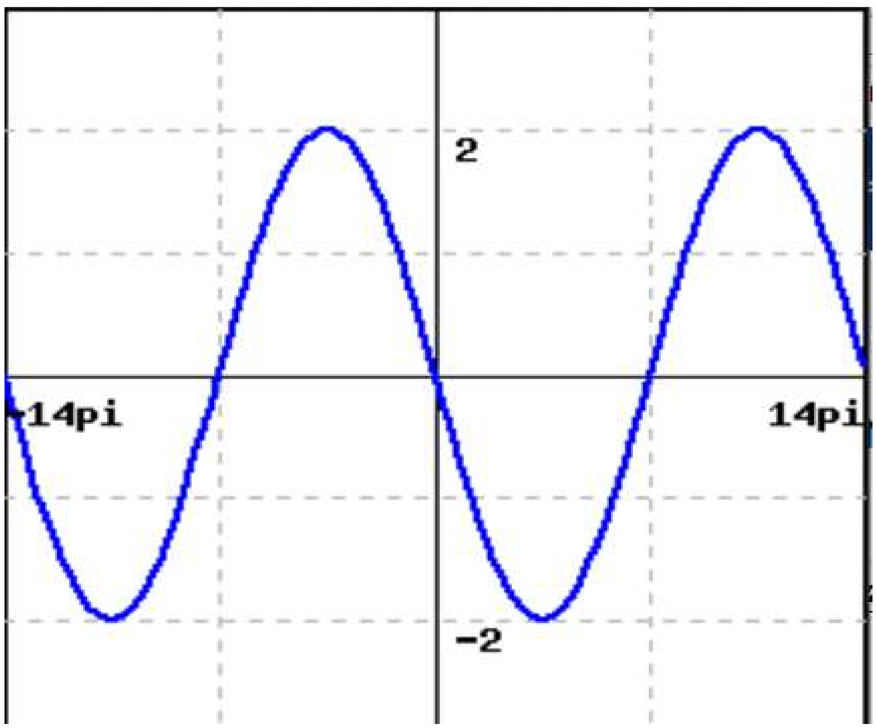

Solution: It's clear that the amplitude is 2, the period $P = 14\pi$, and that the midline is

$$y = 0 \ \ (x\text{-axis})$$

A convenient midline point leads us to choose a sine function. But the graph starts in the middle and goes to low instead of high. It must be a sine graph that is reflected about the x-axis and the multiplier in front must be $a = -2$. There is no horizontal/phase shift ($h = 0$) or vertical shift ($k = 0$) and we have

$$f(x) = -2\sin\left(\frac{2\pi}{14\pi}(x-0)\right) + 0 = -2\sin\left(\frac{x}{7}\right)$$

A quick Desmos/calculator graph with custom window: xmin = -14pi, xmax = 14pi, xscale = 7pi; ymin = -3, ymax = 3, yscale = 1 shows a match with the given graph.