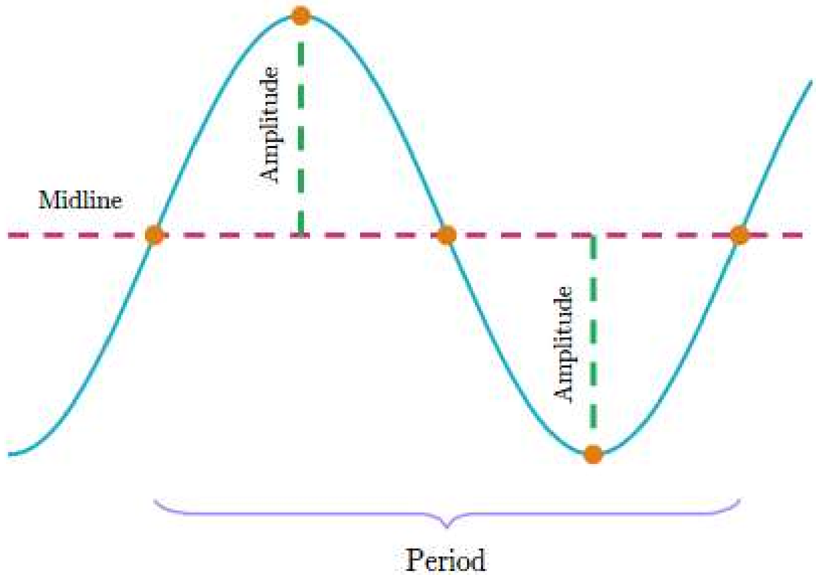
Notes on Graphs of Sine, Cosine Functions

In addition to being periodic, the graphs of the sine and cosine functions have features called **midline** and **amplitude**. Here's an illustration:



When you know the maximum and minimum values of a sine or cosine graph, you can find the amplitude and midline with these formulas:

$$\text{midline equation: } y = \frac{high + low}{2}$$

$$\text{amplitude: } a = \frac{high - low}{2}$$

Note that amplitude is always a **positive** number.

Example: Find the period, midline, and amplitude of a functions whose values are given in this table.

x	-9	-7	-5	-3	-1	1	3	5	7
f(x)	12	10	-4	12	10	-4	12	10	-4

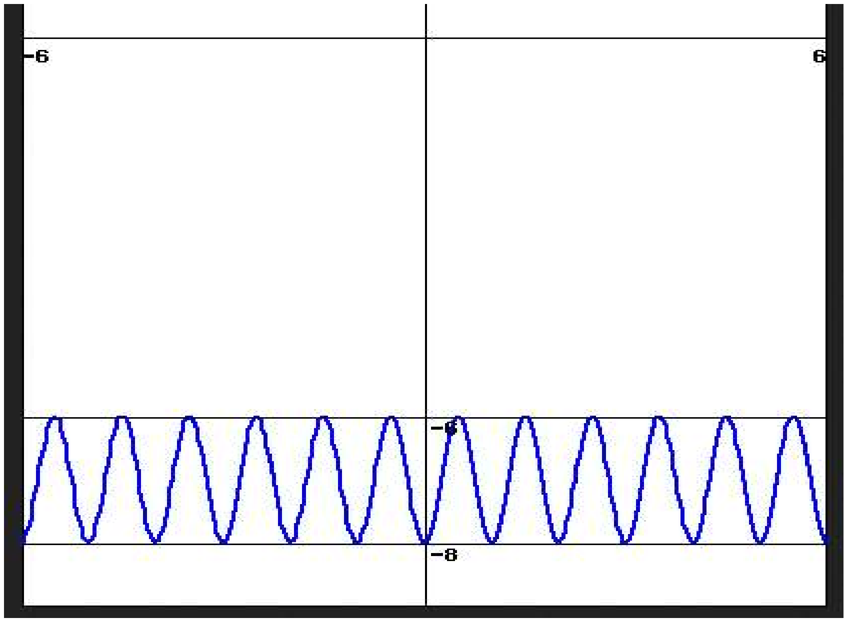
Answers: Tracking output values that repeat, such as 12, the input interval goes from –9 to –3 to 3, an input difference of 6 for successive outputs of 12. The period is 6.

To find the midline and amplitude, use a high value of 12 and a low value of –4. Then

$$\text{midline } y = \frac{12 + (-4)}{2} = 4 \text{ and}$$

$$\text{amplitude } a = \frac{12 - (-4)}{2} = 8$$

Example: Find the midline and amplitude of the function with this graph:



In this graph the high value is -6 and the low value is -8. From this we find that the midline is $y = -7$ and the amplitude $a = 1$.

Graphs of $y = A\sin(B(x-h)) + k$

Using the document

https://faculty.tarleton.edu/jgresham/math1314/transfrm.pdf [copy and paste this link]

on transformations of graphs of functions, we see that the graph of $y = \sin(x)$ undergoes the following changes:

- The **amplitude** is $|A|$, always a positive number, which shows a vertical stretch/squeeze. If A is negative then the graph of $\sin(x)$ is reflected about the x-axis.
- The **period** is $P = \frac{2\pi}{|B|}$, always a positive number, which shows a horizontal stretch/squeeze. If the period P is known from a graph, then $B = \frac{2\pi}{P}$.
- The horizontal shift is h units to the right if $h > 0$ or to the left if $h < 0$. For sine or cosine graphs, h is called the **phase shift**.
- The vertical shift is k units up if $k > 0$ or down if $k < 0$. Therefore the **midline** is $y = k$.

These same features are also found if sin is replaced with cos, the difference being that the cosine graph is a horizontal shift of the sine graph.

To see these features interactively, go to the website

https://www.geogebra.org/m/JDhCNBXU [copy and paste this link]

In this GeoGebra applet, I use c instead of h and d instead of k. Experiment with each slider to see the effect. To reset the sliders click the circling arrows in the upper right corner.

From the graph of $\sin(x)$ whose period is 2π we see that the critical points of the graph–mid, high, low values–occur at the quarter points of the period. The graph starts at $x = 0$ with quarter points $x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ and finishes a cycle at $x = 2\pi$. Here's the general pattern for sine and cosine graphs, and their reflections, with a stretched or squeezed period P:

x	start	$\frac{P}{4}$	$\frac{2P}{4}$	$\frac{3P}{4}$	P
$\sin(x)$	mid	high	mid	low	mid
$-\sin(x)$	mid	low	mid	high	mid
$\cos(x)$	high	mid	low	mid	high
$-\cos(x)$	low	mid	high	mid	low