Notes: Using the Tangent Sum/Difference, Double Angle Formulas

Refer to the Formula List for these examples.

Exercise 1: Use an identity to find the exact value of each expression:
Note: You are not allowed to use decimals in your answer.

$$\frac{\tan(479°) - \tan(239°)}{1 + \tan(479°)\tan(239°)} = ?$$

Solution: This must be the formula for $\tan(A - B)$. Then

$$\begin{aligned}\frac{\tan(479°) - \tan(239°)}{1 + \tan(479°)\tan(239°)} &= \tan(479° - 239°)\\ &= \tan(240°)\\ &= \sqrt{3}\end{aligned}$$

Exercise 2: If $\sin(A) = 0.35$ and $\cos(A) < 0$, determine the following to at least four decimal places:

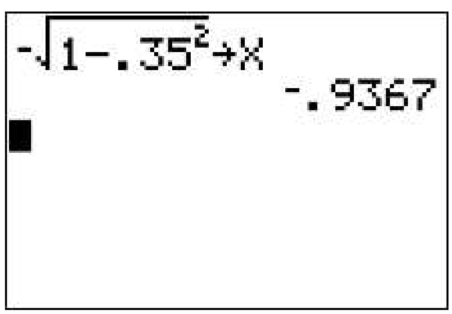
A. $\cos(A) = ?$

Solution: Since $\cos(A)$ is negative we can write

$$\begin{aligned}\cos(A) &= -\sqrt{1 - \sin^2(A)}\\ &= -\sqrt{1 - (0.35^2)}\\ &= -0.9367\end{aligned}$$

Calculator tip: Set your calculator to show four decimal places with **MODE**:

Then store your calculator result in the quick temporary variable **X**:

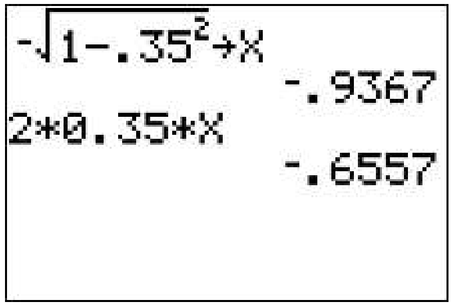


Now whenever we need $\cos(A)$ for the calculator we can press **X**, which still has the full internal accuracy for the result.

B. $\sin(2A) = ?$

Solution: Use the double angle formula for sine. On the calculator type **X** for $\cos(A)$

$$\begin{aligned}\sin(2A) &= 2\sin(A)\cos(A)\\ &= 2(.35)\left(-.9367\ [\text{use X}]\right)\\ &= -0.6557\end{aligned}$$



C. $\cos(2A) = ?$

Solution: Your have a choice of two formulas. Use the one with the **original** information

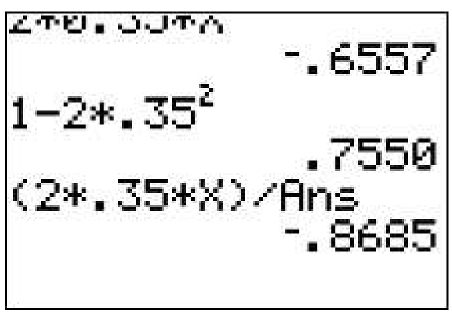
$$\begin{aligned}\cos(2A) &= 1 - 2\sin^2(A)\\ &= 1 - 2 * (.35)^2\\ &= 0.7550 \ \ \text{(this is exact)}\end{aligned}$$

D. $\tan(2A) = ?$

Solution: Use the Quotient Identity

$$\begin{aligned}\tan(2A) &= \frac{\sin(2A)}{\cos(2A)}\\ &= \frac{2\sin(A)\cos(A)}{1 - 2\sin^2(A)}\\ &= \frac{2 * (.35) * \left(-.9367 \text{ use X}\right)}{1 - 2 * (.35)^2}\\ &= -0.8685\end{aligned}$$

We can use the **ANS** variable on the calculator, which now stores 0.7550, to help the setup.



D. The quadrant for $2A$ is ?

Solution. Note that $\sin(2A)$ is negative and $\cos(2A)$ is positive. The angle $2A$ must be in QIV.

Exercise 3: If $\tan\theta = 9/6$ and $\cos\theta > 0$ then find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$.

Solution: We must find $\sin\theta$ and $\cos\theta$. Since tangent and cosine are positive, θ is in QI. Then

$$\tan\theta = \frac{y}{x} = \frac{9}{6} = \frac{3}{2}$$

we have

$$x = 2$$
$$y = 3$$
$$r = \sqrt{2^2 + 3^2} = \sqrt{13}$$

Then

$$\sin\theta = \frac{3}{\sqrt{13}}$$
$$\cos\theta = \frac{2}{\sqrt{13}}$$

Now use the double angle formulas:

$$\begin{aligned}\sin 2\theta &= 2\sin\theta\cos\theta \\ &= 2\left(\frac{3}{\sqrt{13}}\right)\left(\frac{2}{\sqrt{13}}\right) \\ &= \frac{12}{13}\end{aligned}$$

and

$$\begin{aligned}\cos 2\theta &= 2\cos^2\theta - 1 \\ &= 2\left(\frac{2}{\sqrt{13}}\right)^2 - 1 \\ &= 2\left(\frac{4}{13}\right) - \frac{13}{13} \\ &= -\frac{5}{13}\end{aligned}$$

Since we know both sine and cosine of 2θ exactly, it is simplest to use the Quotient Identity to find $\tan 2\theta$

$$\begin{aligned}\tan 2\theta &= \frac{\sin 2\theta}{\cos 2\theta} \\ &= \frac{\frac{12}{13}}{-\frac{5}{13}} \\ &= -\frac{12}{5}\end{aligned}$$

Exercise 4: Find $\cos 2\theta$ if $\sin\theta = \frac{8}{17}$.

Solution: Use the cosine double angle formula that has the sine function:

$$\begin{aligned}\cos 2\theta &= 1 - 2\sin^2\theta \\ &= 1 - 2\left(\frac{8}{17}\right)^2 \\ &= \frac{161}{289}\end{aligned}$$

Don't forget the calculator tricks to help with fractions.

Comment In these exercises we found the sine and cosine double angle before finding the tangent double angle. In such cases it's easiest to use the Quotient Identity

$$\tan(2A) = \frac{\sin(2A)}{\cos(2A)}$$

If we're given the value of $\tan A$ and nothing else, we can still find $\tan 2A$ with the tangent double angle formula.

Example: Given: $\tan A = \frac{3}{4}$. Find $\tan 2A$.

Solution: There's not enough information to determine whether and A is in QI or QIII, and so we cannot find $\sin A$ or $\cos A$. But we can still answer the question

$$\begin{aligned}\tan 2A &= \frac{2\tan A}{1-\tan^2 A} \\ &= \frac{2 * \left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2} \\ &= \frac{24}{7}\end{aligned}$$

Score an assist to the calculator.