Solving Trig Equations Part 1

Since trig functions are periodic, an equation with trig functions will generally have infinitely many solutions. In most problems you will be asked to find all solutions in the interval $[0, 2\pi]$ (radian mode) or $[0°, 360°]$ (degree mode).

The technique used in these problems is to reduce the equation to the form

$$trigfunction(angle) = number$$

The inverse of the trig function will give you one answer, but there may be solutions in other quadrants which can be found using reference angles.

To help you see what is going on with the solutions, we will solve these equations in degree mode, and then convert the answer(s) to radians if required. A graph may also be helpful.

Example: Solve the following equation in the interval $[0, 2\pi]$.

$$\sin(t) = -\frac{\sqrt{3}}{2}$$

Solution: This problem requires answers in radian mode. Since the sine function is negative in QIII and QIV we must look for answers in those quadrants. We first find the **reference angle** for the solutions. Remove the minus sign and find

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$$

The angle $60°$ is NOT the solution. It is the *reference angle* for the two solutions, one in QIII and the other in QIV.

$$t = 240°, 300°$$

Now convert these angles to radians to obtain

$$t = \frac{4}{3}\pi, \frac{5}{3}\pi$$

The format of the answer in the exercises may be

$$\boxed{4/3,5/3}\,\pi$$

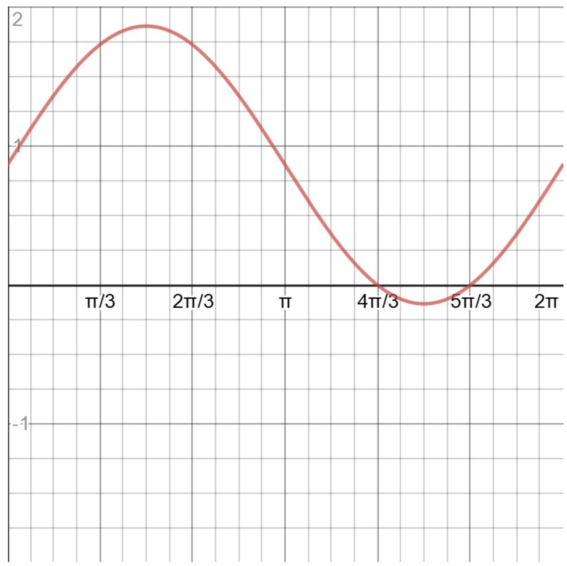
To visualize this result we will graph the equation using

$$y = leftside - rightside$$

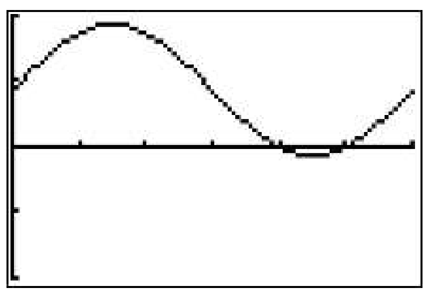
and then look for the x-intercepts. Here we use

$$\sin\left(x - \left(-\frac{\sqrt{3}}{2}\right)\right) = \sin\left(x + \frac{\sqrt{3}}{2}\right)$$

On Desmos, use $0 \le x \le 2\pi$ Step: $\frac{\pi}{3}$ and $-2 \le y \le 2$ Step:1



You can see that the graph crosses the *x*-axis at the solution points. You can also check this with a calculator graph using radian mode and the same window settings.



Here you can see that the graph crosses the x-axis at the $4th$ and $5th$ tick marks where each mark is a multiple of $\frac{\pi}{3}$.

Example: Solve the following equation in the interval $[0, 2\pi]$.

$$\cos(t) = -\frac{\sqrt{3}}{2}$$

Solution: Since cosine is negative in QII and QIII, we will find the reference angle for the solutions. Remove the minus sign and find

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$$

The angle $30°$ is NOT the solution. It is the *reference angle* for the two solutions, one in QIII and the other in QIV.

$$t = 150°, 210°$$

Now convert these angles to radians to obtain

$$t = \frac{5}{6}\pi, \frac{7}{6}\pi$$

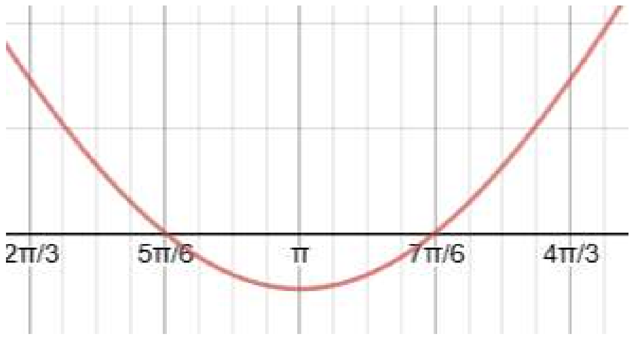
The format of the answer in the exercises may be

$$\boxed{5/6,7/7}\,\pi$$

You should set up a graph of

$$y = \cos(x) + \frac{\sqrt{3}}{2}$$

and the same Window but using Xscale=$\pi/6$. Here's a snip showing the solutions:



Example: Find all exact solutions to the equation

$$\tan(t) = \frac{1}{\tan(t)}$$

in the interval $0 \le t \le 2\pi$. Enter your answers as a comma separated list.

Solution: Multiply both sides of the equation by $\tan(t)$ to obtain

$$\tan^2(t) = 1$$

Then use the square root method to solve this quadratic equation in $\tan(t)$.

$$\tan t = \pm\sqrt{1}$$

This gives us two separate equations to solve

$$\left[\ \tan(t) = 1 \quad \text{or} \quad \tan(t) = -1\ \right]$$

In either equation the reference angle is $45°$. In the first equation the solutions are in Q1 and QIII (tangent is positive).

$$t = 45°, 225°$$

For the second equation, the solutions are in QII and QIV (tangent is negative)

$$t = 135°, 315°$$

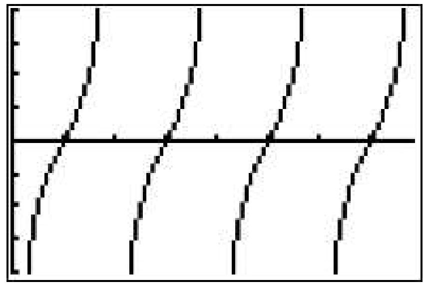
Convert these to radians.

$$t = \pi/4, 5\pi/4, 3\pi/4, 7\pi/4$$

Here's a calculator graph using radian mode

Y1=tan(X)-1/tan(X)

and window: Xmin = 0 Xmax = 2π Xscl = $\pi/4$ Ymin = -4 Ymax = 4 Yscl = 1



Note that the graph crosses the x-axis at the 1st, 3rd, 5th, and 7th tick marks where each mark represents a multiple of $\frac{\pi}{4}$.

Example: Use trigonometric identities to solve $\sin(2\theta) - \cos(\theta) = 0$ exactly for $0 \le \theta < 2\pi$. If there is more than one answer, enter your answers as a comma separated list.

Solution: We will use the sine double angle identity and then factor the left side.

$$\sin(2\theta) - \cos(\theta) = 0$$
$$2\sin(\theta)\cos(\theta) - \cos(\theta) = 0$$
$$\cos(\theta)(2\sin(\theta) - 1) = 0$$

Now set each factor equal to 0 and solve

$$\begin{bmatrix} \cos(\theta) = 0 & \text{or} & 2\sin(\theta) - 1 = 0 \\ \theta = 90°, 270° & & \sin(\theta) = \frac{1}{2} \\ & & \theta = 30°, 150° \end{bmatrix}$$

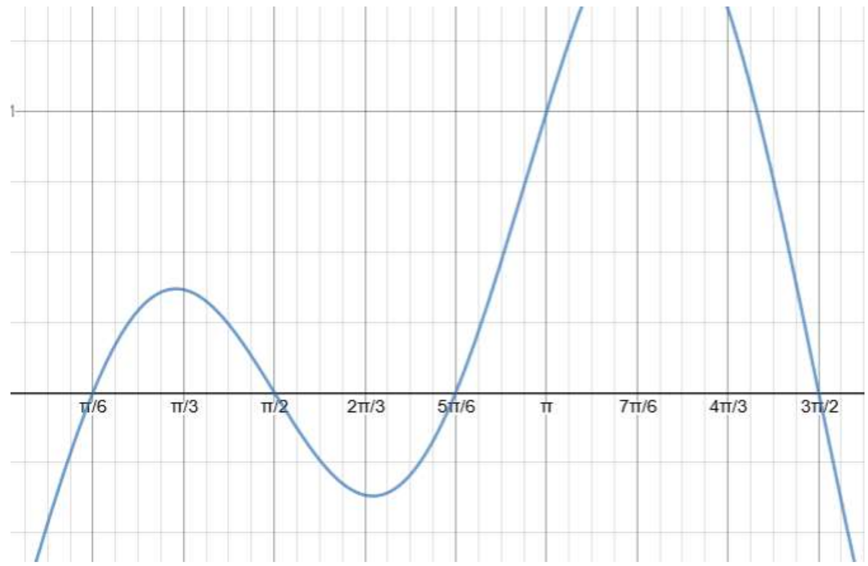
A unit circle may be useful here. All these exact answers must now be converted to radians.

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$

To visualize these solutions, graph

$$y = \sin(2x) - \cos(x)$$

(Desmos uses x) with Xscl = $\frac{\pi}{6}$ and look for the x-intercepts.


1
π/6
π/3
π/2
2π/3
5π/6
π
7π/6
4π/3
3π/2