Notes on Modeling with Sine and Cosine Functions

When you encounter a periodic function, perhaps described as **seasonal**, you will be setting up a sinusoid of the form

$$f(x) = A\sin(B(x-h)) + k$$

or

$$f(x) = A\cos(B(x-h)) + k$$

If you have information that describes values at a midline, you should consider a sine function. If you have information that describes values at a high or low point, you should consider a cosine function. Here's two examples.

Example Over the past several years, the owner of a boutique on Aspen Avenue has observed a pattern in the amount of revenue for the store. The revenue reaches a maximum of about \$ 41000 in April and a minimum of about \$ 26000 in October. Suppose the months are numbered 1 through 12, and write a function of the form $f(x) = A\sin(B[x-C]) + D$ that models the boutique's revenue during the year, where x corresponds to the month.

Solution: Since we are given maximum and minimum values, \$41K, and \$26K (units of thousands to simplify the numbers), we should model this with a cosine function (contrary to the instructions in the exercise):

$$f(x) = A\cos(B(x-h)) + k$$

From the high and low values we can calculate the midline

$$y = k = \frac{41000 + 26000}{2} = 33500$$

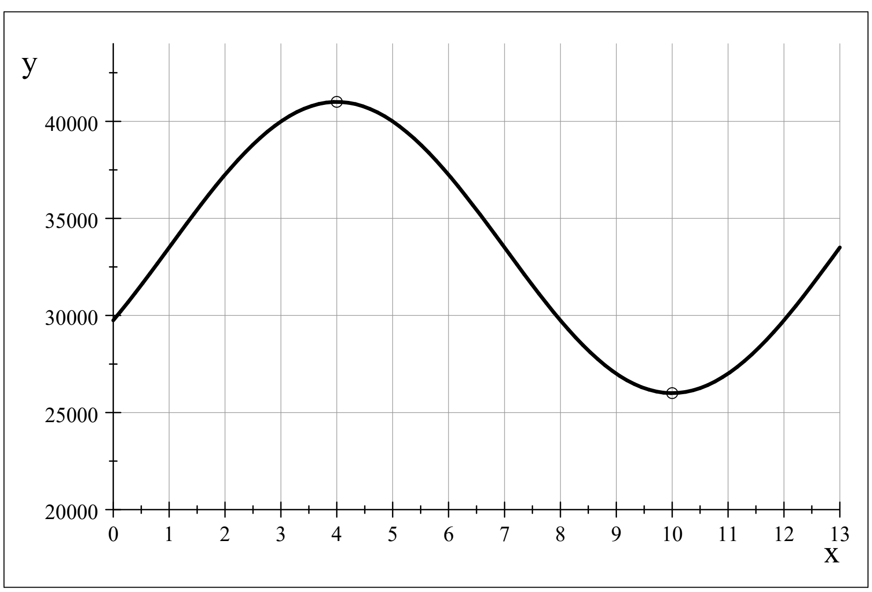
and the amplitude

$$A = \frac{41000 - 26000}{2} = 7500$$

The period is 12 (months) since these seasonal values occur repeatedly in April (month 4) and October (month 10). Since the high value is at month 4 instead of month 0 (the cosine graph normally has a high value at $x = 0$), there is a horizontal shift of +4. This leads to using a multiplier $+A$:

$$f(x) = 7500\cos\left(\frac{2\pi}{12}(x-4)\right) + 33500$$

Here's a graph with the key points marked. Note that month 0 = month 12 = December.



Example: A ferris wheel is 160 meters in diameter and boarded at its lowest point (6 O'Clock) from a platform which is 8 meters above ground. The wheel makes one full rotation every 16 minutes, and at time $t = 0$ you are at the loading platform (6 O'Clock). Let $h = f(t)$ denote your height above ground in meters after t minutes. Give units with each part.

Solution:

1. **A**. First note that the **period**, the time for one rotation, is 16 minutes, written **16min**

B. The minimum height is 8 meters and maximum is $8 + 160 = 168$ meters so that the **midline**

$$y = k = \frac{168 + 8}{2} = 88$$

Enter **88m**

C. The **amplitude** is

$$A = \frac{168 - 8}{2} = 80$$

Enter **80m**

D. We are asked to match a graph representing two full rotations, that is from $t = 0$ to $t = 32$ (twice the period). Let's get the function and graph it. Since we're starting at a low point, we should use a negative multiplier $-A$ with a cosine function [low, med, high, med, low]. The phase shift $h = 0$ since we're at the low point at time zero.

$$f(t) = -80\cos\left(\frac{2\pi}{16}(t - 0)\right) + 88$$

Here's the graph for this version of the problem. We're looking for $0 \le t \le 32$ (two periods) and $0 \le y \le 168$ to show the max height: