Notes on Vectors: Examples of Vector Operations Part 1

For these examples of vector operations refer to the summary notes

https://faculty.tarleton.edu/jgresham/Math%20109/notes_vector.pdf [copy and paste this link]

Also refer to the three **illustrators** found in the Math 1316 notes at

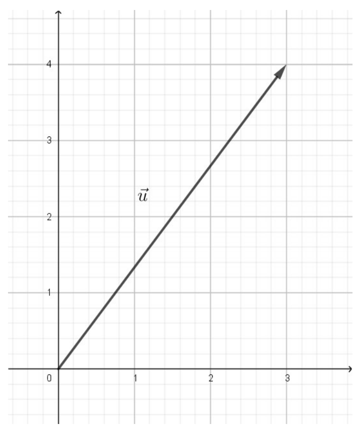
https://faculty.tarleton.edu/jgresham/Math%20109/math-109.html

In this part we consider conversion of vector forms between rectangular and polar form. Polar form provides a good visualization of a vector whereas rectangular form provides an easy way to add vectors.

When we write vectors we use angle brackets, not parentheses. The notation $(3,4)$ refers to the specific point on the x-y plane, but **angle brackets** $\vec{u} = \langle 3,4 \rangle$ is a vector with a horizontal displacement of 3 and a vertical displacement of 4. When you write this vector for online problems you will use the **less than** and **greater than** symbols:

$$\mathbf{u} =< 3,4 >$$

Notice that the vector naming notation is either the vector name with an arrow above it or the vector name in **bold** print.
Here's a picture of this vector.



This vector is drawn with beginning point $(0,0)$, but the same vector could be drawn with any other beginning point. If we consider this vector in polar form we get its **magnitude** (or *length* or *absolute value*) is

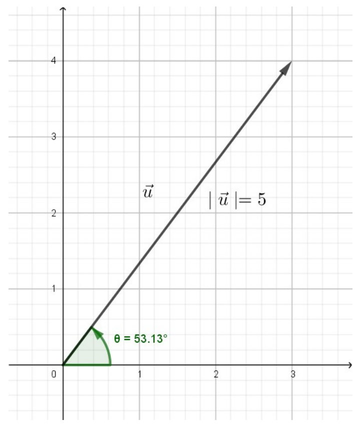
$$|\mathbf{u}| = \sqrt{3^2 + 4^2} = 5$$

and its **angle** θ is

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) \cong 53.3\,°$$

Then

$$\mathbf{u} = \langle 3,4 \rangle = \langle 5\angle 53.3\,° \rangle$$



Example 1: Convert polar form to rectangular form: $\langle 6\angle 300°\rangle$.

Solution: Use the conversion formula $\langle r\cos\theta, r\sin\theta\rangle$ to get

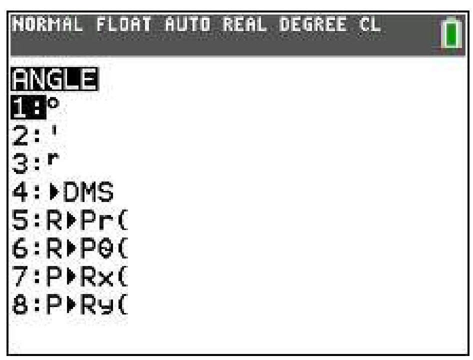
$$\langle 6\cos 300°, 6\sin 300°\rangle$$
$$= \left\langle 6\left(\tfrac{1}{2}\right), 6\left(-\tfrac{\sqrt{3}}{2}\right)\right\rangle$$
$$= \left\langle 3, -3\sqrt{3}\right\rangle$$

so that

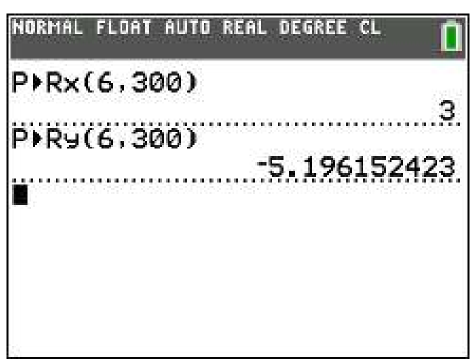
$$x = 3 \quad \text{and} \quad y = -3\sqrt{3}$$

For y type: -3sqrt(3)

Calculator Trick: Press <2ND><ANGLE> on the TI-83 or -84 to bring up the ANGLE menu:



Then select 7. P->Rx(and enter 6,300) [degree mode here!] and press <ENTER>.
Repeat these steps with 8. P->Ry(6,300).

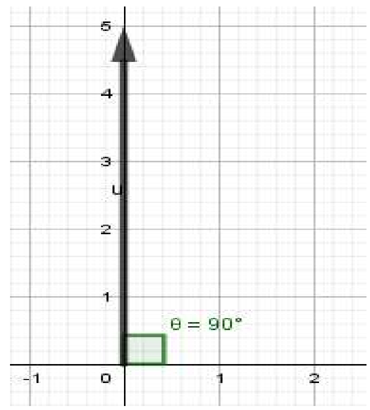


The result for y is the decimal version of $-3\sqrt{3}$. An exact answer, if possible, is preferred in these conversions.

Example 2: Convert the following rectangular coordinates into polar coordinates. Always choose $0 \le \theta < 360°$

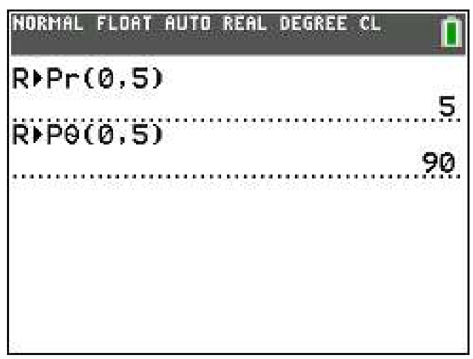
1. **A**. $\langle 0, 5 \rangle$.

Solution: It may be helpful to sketch a picture of this vector.



The length of this vector is 5. But since $\langle a, b \rangle = \langle 0, 5 \rangle$, $\tan\theta = \frac{b}{a} = \frac{5}{0}$ is undefined, we cannot find the angle with $\tan^{-1}(\frac{b}{a})$. The tip of the vector lies on the positive x-axis and so $\theta = 90°$.

Calculator Trick: <2ND><ANGLE> and use 5. R->Pr(and then again 6. R->Pθ(



B. $\left\langle -\sqrt{3}, -1 \right\rangle$

Solution: To find $r = \sqrt{a^2 + b^2}$ substitute

$$\begin{aligned} r &= \sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2} \\ &= \sqrt{3 + 1} \\ &= 2 \end{aligned}$$

To find θ we note that

$$\tan\theta = \frac{-1}{-\sqrt{3}} = \frac{\sqrt{3}}{3} \text{ and } \theta \text{ is in QIII}$$

On the calculator

$$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = 30°$$

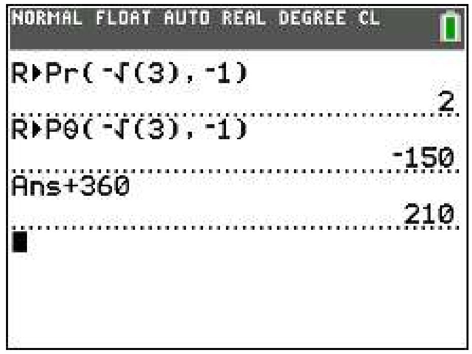
but this is not angle θ. It is the **reference angle** for θ in QIII. Then

$$\theta = 180° + 30° = 210°$$

Then we have

$$r = 2 \quad \text{and} \quad \theta = 210°$$

Now let's look at the calculator version:



Notice that the R->Pθ(-$\sqrt{3}$, -1) gave a correct angle, but expressed as a negative angle. If the angle is in QIII or QIV, the answer will be a negative angle. Just add 360 degrees to insure that it is between $0°$ and $360°$ as the problem requires.

Example 3: A bullet is fired into the air with an initial velocity of 900 feet per second at an angle of $59°$ from the horizontal. The horizontal and vertical components of the velocity vector are ?

Solution: Calculate

$$< 900\cos 59°, 900\sin 59° >$$
$$=< 463.5343, 774.4506 >$$

Example 4: Find the vector of length 2 making an angle of $30°$ with the x-axis.

Solution:

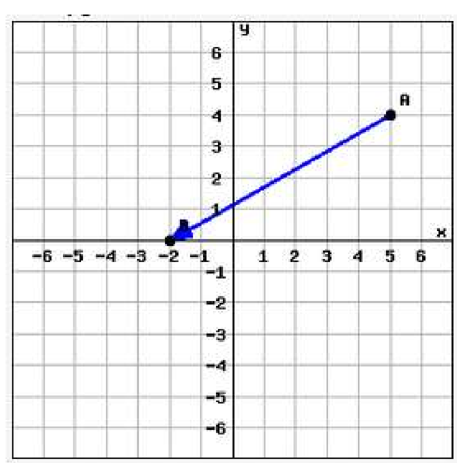
$$\langle 2\angle 30° \rangle = \langle 2\cos 30°, 2\sin 30° \rangle$$
$$= \left\langle \sqrt{3}, 1 \right\rangle$$

Use less than and greater than symbols to express the angle online

<sqrt(3),1>

Don't forget the < > or it will be counted wrong. Also ( ) will be incorrect.

Example 5: Find the vector $\overrightarrow{AB}$ in R^2 given in the figure.



Solution: The vector starts at $A(5,4)$ and ends at $B(-2,0)$. Use $B - A$ by subtracting coordinates:

$$\overrightarrow{AB} = \langle -2-5, 0-4 \rangle = \langle -7, -4 \rangle$$

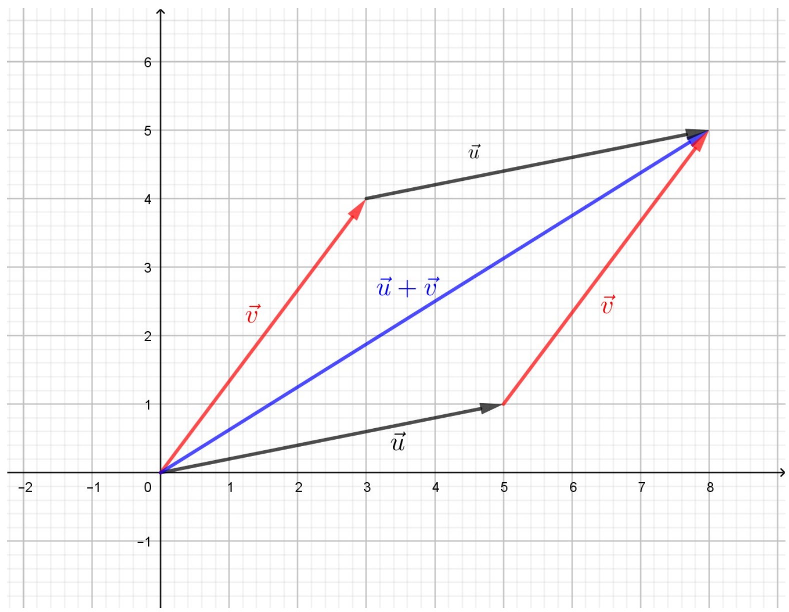
Type <-7,-4> for an online answer.

Example 6: Add vectors: $\langle 5,1 \rangle + \langle 3,4 \rangle$.

Solution: To add vectors in rectangular form, just add the corresponding components:

$$\langle 5,1 \rangle + \langle 3,4 \rangle = \langle 5+3, 1+4 \rangle$$
$$= \langle 8,5 \rangle$$

Here's a picture that illustrates this problem



In this we see that the sum vector is the diagonal of the parallelogram. It is also the **resultant** (another name for **sum**) that comes from placing the tail of $\vec{v}$ at the tip of $\vec{u}$ (tip-to-tail addition).

Vector addition is used in physics to add forces, and to add displacements. We will give examples in the notes on vector applications.

Exercise 7: Calculate:

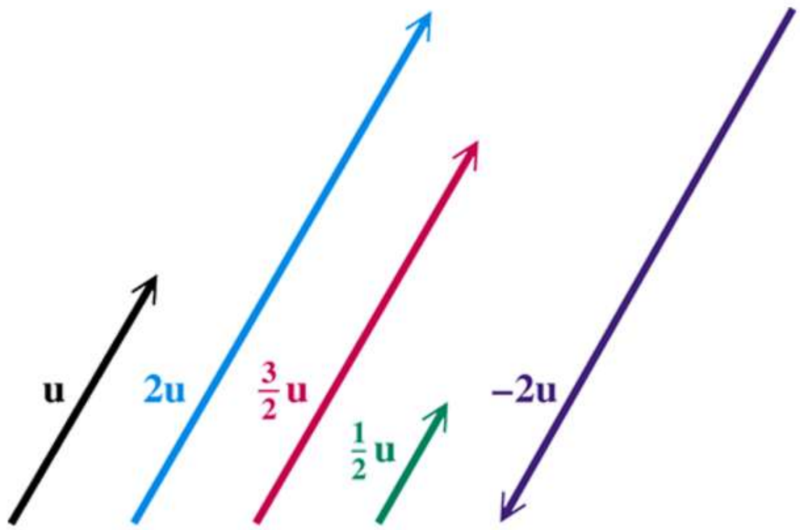
$$3\langle 2,4 \rangle$$

Solution: This operation is called *scalar multiplication.* Just multiply the scalar, 3, by each component.

$$3\langle 2,4 \rangle = \langle 6,12 \rangle$$

The vectors $\langle 2,4 \rangle$ and $\langle 6,12 \rangle$ both point in the same direction (they are parallel), but the length of $\langle 6,12 \rangle$ is three times the length of $\langle 2,4 \rangle$.

Here's some illustrations of scalar multiplicaton



[Figure from the Math 1316 textbook Trigonometry, 11th Edition, authors Margaret Lial, JohnHornsby, David I. Schneider, Callie Daniels. Published by Pearson]

Exercise 8: Suppose $\vec{u} = \langle 1,5 \rangle$ and $\vec{v} = \langle 3,4 \rangle$. Find the following:

1. **A.** $\vec{u} - \vec{v} = ?$

Solution: Instead of adding components, subtract them:

$$\begin{aligned} \vec{u} - \vec{v} &= \langle 1-3, 5-4 \rangle \\ &= \langle -2, 1 \rangle \end{aligned}$$

B. $-\frac{1}{3}\vec{v} = ?$

Solution:

$$-\frac{1}{3}\vec{v} = -\frac{1}{3}\langle 3,4 \rangle = \left\langle -1, -\frac{4}{3} \right\rangle$$

C. $5\vec{u} - 7\vec{v} = ?$

Solution: Do the scalar multiplication, and then subtract:

$$\begin{aligned} 5\vec{u} - 7\vec{v} &= 5\langle 1,5 \rangle - 7\langle 3,4 \rangle \\ &= \langle 5,25 \rangle - \langle 21,28 \rangle \\ &= \langle -16,-3 \rangle \end{aligned}$$