Notes and Examples for Math 109 Chapter 2 Section 1

Right-Triangle-Based Definitions

In this chapter we will consider only acute angles, their trig functions, and using the trig functions to solve right triangles. In our text on page 46 is this picture of the right-triangle definitions of the trig functions of angle *A*.

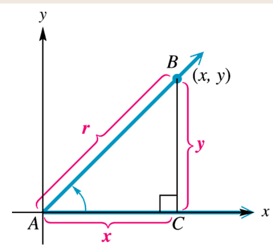
Right-Triangle-Based Definitions of Trigonometric Functions

Let *A* represent any acute angle in standard position.

$$\sin A = \frac{y}{r} = \frac{\text{side opposite } A}{\text{hypotenuse}} \qquad \csc A = \frac{r}{y} = \frac{\text{hypotenuse}}{\text{side opposite } A}$$

$$\cos A = \frac{x}{r} = \frac{\text{side adjacent to } A}{\text{hypotenuse}} \qquad \sec A = \frac{r}{x} = \frac{\text{hypotenuse}}{\text{side adjacent to } A}$$

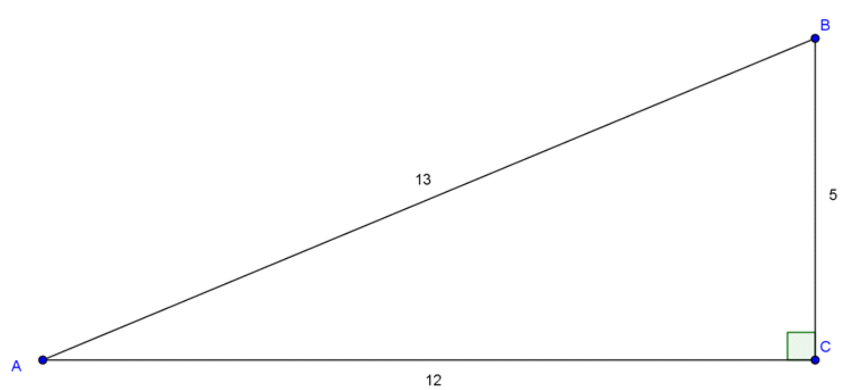
$$\tan A = \frac{y}{x} = \frac{\text{side opposite } A}{\text{side adjacent to } A} \qquad \cot A = \frac{x}{y} = \frac{\text{side adjacent to } A}{\text{side opposite } A}$$



 4

There is a mnemonic for this: sohcahtoa. [**s**ine is **o**pposite over **h**ypotenuse, etc.]

Example   In this example, note that angles *A* and *B* are **complementary**, that is, $A + B = 90°$.



We have

$$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{5}{13}$$

$$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{12}{13}$$

$$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{5}{12}$$

$$\cot A = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{12}{5}$$

Note that the tangent and cotangent are reciprocals of each other.
If we consider angle *B*, we have

$$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{12}{13} = \cos A$$

$$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{5}{13} = \sin A$$

$$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{12}{5} = \cot A$$

Cofunction Rule

The previous example also illustrates the
Cofunction Rule: If we use the word *fcn* to represent the functions sin, tan, or sec and if A and B are acute angles with $A + B = 90°$, then

$$fcn A = cofcn B$$

This rule is found on page 47 in the textbook.

Example We can use the cofunction rule to write trig functions of angles in terms of their cofunctions:

$$\sin 60° = \cos(90° - 60°) = \cos 30°$$

In older trig textbooks which have tables, the tables are written for angles from $0°$ to $45°$. The tables are arranged to use the Cofunction Rule to find the trig function values for angles between $45°$ and $90°$.

Increasing and decreasing functions for angles in quadrant I

If angle A is between $0°$ and $90°$, as A increases, the sin, tan, and sec function output values of A also increase. But the **co**functions cos, cot, and csc values *decrease.*

See page 48 in the text for the summary illustration, and use the web site

http://faculty.tarleton.edu/jgresham/geogebra/trig_defs.html

to see an interactive demonstration for sine, cosine, tangent.

Examples

1. True or False: $\sin 23° > \sin 27°$

Answer: False. Since $23° < 27°$ and the sine function is increasing, we should have $\sin 23° < \sin 27°$

2. True or False: $\cos 23° > \cos 27°$

Answer: True. The cosine function is *decreasing*. If the angle gets larger, the output value of cosine of the angle gets smaller.

3. True or False: $\sin 47° < \cos 57°$

Answer: False. We must use the Cofunction Rule to express both sides of the inequality with the same trig function:

$$\begin{aligned}\sin 47° &< \cos 57° \\ &= \sin(90° - 57°) \\ &= \sin 43°\end{aligned}$$

Since $47° > 43°$ and sine is increasing, we should have

$$\sin 47° > \sin 43°$$

Trig functions of the special angles $30°$, $60°$, and $45°$

In addition to the quadrantal angles $0°,\ 90°,\ 180°,\ 270°,$ and $360°$ these angles are special because they lead to geometric isosceles and equilateral triangles. The development of these trig function values is outlined in our text on pages 49 and 50. These angle have trig function output values that can be expressed *exactly* in terms of fractions and radicals. To help you remember these, here is a version of the chart on page 50 which contains mnemonic patterns for sine and cosine

θ	$\sin\theta$	$\cos\theta$
$0°$	$\frac{\sqrt{0}}{2} = 0$	$\frac{\sqrt{4}}{2} = 1$
$30°$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$45°$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$60°$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$
$90°$	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sqrt{0}}{2} = 0$

Notice that the Cofunction Rule can be used to get the output values for the trig functions of these special angles: $\cos 0° = \sin 90°;\ \cos 30° = \sin 60°;\ \cos 45° = \sin 45°;$ etc.

The tangent function output values on page 50 can then be developed using the quotient identity

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

θ	$\tan\theta$
$0°$	$\dfrac{\sin 0°}{\cos 0°} = \dfrac{0}{1} = 0$
$30°$	$\dfrac{\sin 30°}{\cos 30°} = \dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$
$45°$	$\dfrac{\sin 45°}{\cos 45°} = \dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$
$60°$	$\dfrac{\sin 60°}{\cos 60°} = \dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$
$90°$	$\dfrac{\sin 90°}{\cos 90°} = \dfrac{1}{0} =$ *undefined*

Example:

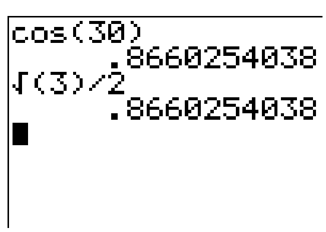
Find the exact value of $\sec 30°$.

Answer: Since this function is not in our table on page 50, we must use the reciprocal identity $\sec\theta = \dfrac{1}{\cos\theta}$.

$$\begin{aligned} \sec 30° &= \frac{1}{\cos 30°} \\ &= \frac{1}{\frac{\sqrt{3}}{2}} \\ &= \frac{2}{\sqrt{3}} \\ &= \frac{2\sqrt{3}}{3} \end{aligned}$$

So that $\sec 30° = \dfrac{2\sqrt{3}}{3}$.

The calculator can be used to get 10-significant-digit approximations to support these output values. Here's an example.



This is **not mathematical proof**. But it is strong evidence. Since there are infinitely many decimal places, it *could* be the case that the numbers differ in an unseen decimal place (not likely!).