Complex Numbers

Definition of *i*

$$i = \sqrt{-1} \text{ and } i^2 = -1$$

With this definition, we can handle square roots of negative numbers:

For $x > 0$, $\sqrt{-x} = \sqrt{-1}\sqrt{x} = i\sqrt{x}$

Example $\sqrt{-25} = 5i$

Definition *A complex number can be written in the form $a + bi$ where a and b are real numbers, and $i = \sqrt{-1}$.*

Definition *The conjugate of a complex number $a + bi$ is defined as*

$$\overline{a+bi} = a - bi$$

Example

$$\overline{2-3i} = 2 + 3i$$

Operations with complex numbers

1. Complex numbers can be *added* or *subtracted* by combining like terms.
2. Complex numbers can be *multiplied* by the *FOIL METHOD*.
3. Complex numbers can be *divided* by multiplying numerator and denominator by the *conjugate of the denominator*.

Example *Add:* $(3 + 2i) + (4 - 8i) = 7 - 6i$

Example *Subtract:* $(5 - 2i) - (2 - 6i) = 3 + 4i$

Example *Multiply:* $(5 - 3i)(4 + 2i)$

Solution *Use the FOIL method to obtain*

$$\begin{aligned}(5-3i)(4+2i) &= 20 + 10i - 12i - 6i^2 \quad \textit{FOIL} \\ &= 20 - 2i - (6)(-1) \quad \textit{replace } i^2 \textit{ with } -1 \\ &= 20 - 2i + 6 \\ &= 26 - 2i\end{aligned}$$

Example *Divide:* $\dfrac{26-2i}{4+2i}$

Solution *Multiply numerator and denominator by* $4 - 2i$

$$\begin{aligned}\frac{26-2i}{4+2i} &= \frac{26-2i}{4+2i} \cdot \frac{4-2i}{4-2i} \quad \textit{conjugate of denominator} \\ &= \frac{104 - 52i - 8i + 4i^2}{16 - 8i + 8i - 4i^2} \\ &= \frac{104 - 60i - 4}{16 + 4} \\ &= \frac{100 - 60i}{20} \\ &= 5 - 3i\end{aligned}$$