Examples of Polynomial Function Graphs

$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$

The leading coefficient $a_n \neq 0$ The constant term a_0 is the y-intercept.

1. **Degree 0** (**Constant**) $y = a_0$. Graph is a **horizontal straight line** through $(0, a_0)$.

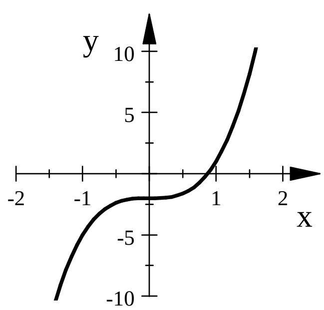
2. **Degree 1** (**Linear**) $y = a_1 x + a_0$. Graph is a **straight line** with slope a_1 and y-intercept $(0, a_0)$.

3. **Degree 2** (**Quadratic**) $y = a_2 x^2 + a_1 x + a_0$. Graph is a **parabola** which opens upward if $a_2 > 0$, downward if $a_2 < 0$.

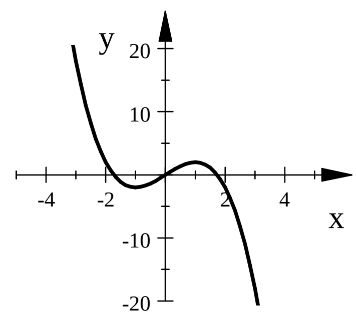
4. **Degree 3** (**Cubic**) $y = a_3 x^3 + a_2 x^2 + a_1 x + a_0$. The graph will move low-to-high ($a_3 > 0$) or high-to-low ($a_3 < 0$) with one *inflection point* and either two or no turning points.

   A. Examples:

   $y = 3x^3 - 2$



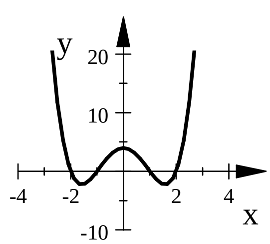
   $y = -x^3 + 3x$



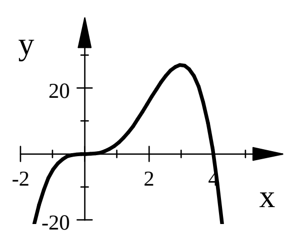
   Confirm these graphs with your calculator, being sure to note the graphing window for each.

5. **Degree 4** (**Quartic**) $y = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$. The graph of a quartic will either open upward ($a_4 > 0$) or downward ($a_4 < 0$) with either 1 or 3 turning points. There a several general types of possibilities for this, and here's some examples.

   A. $y = x^4 - 5x^2 + 4$



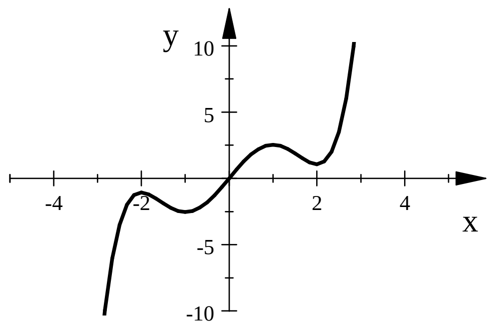
   $y = -x^4 + 4x^3$



6. **Degree 5 (Quintic)** $y = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$

A. As in the degree 1 and degree 3 polynomial, the graph of the quintic will move low-to-high ($a_5 > 0$) or high-to-low ($a_5 < 0$) with up to 4 turning points. Try this example with your calculator and the viewing window.

$y = \frac{1}{5}x^5 - \frac{5}{3}x^3 + 4x$



General patterns for the graphs of polynomial functions

On a large scale, the graph of a polynomial of degree n will resemble the graph of $y = a_nx^n$.

1. Odd Degree polynomial

In general, the graphs of polynomials of odd degree will move from $-\infty$ to $+\infty$ (low-to-high if $a_n > 0$) or $+\infty$ to $-\infty$ (high-to-low if $a_n < 0$).

2. Even Degree polynomial

Graphs of polynomials of even degree will open upward (both ends up if $a_n > 0$) or open downward (both ends down if $a_n < 0$).

3. Turning points

A polynomial of degree n will have, **at most**, $n - 1$ turning points. The turning points may not be visible on a large scale.

Graphing tips A zoom decimal window, though it gives the nicest trace values, rarely shows the complete behavior of polynomials of degree 3 or larger. Be prepared to adjust the window to show all possible turning points and end behavior. In some cases, no single window will effectively show all turning points and end behavior.