Systems of Equations

A **solution of a system** of equations in two variables is an **ordered pair** (x, y) of real numbers, or a **point** in a two-dimensional coordinate system. A solution point is at an intersection of the graphs of the equations in the system.

Systems of Linear Equations

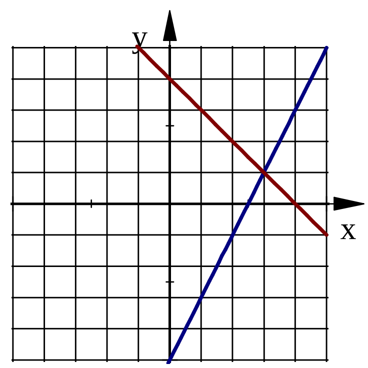
1. Two linear equations in two variables

A. Types of solution sets

i. A single point

The two lines cross at a single point

$$\begin{cases} 2x - y = 5 \\ x + y = 4 \end{cases}$$

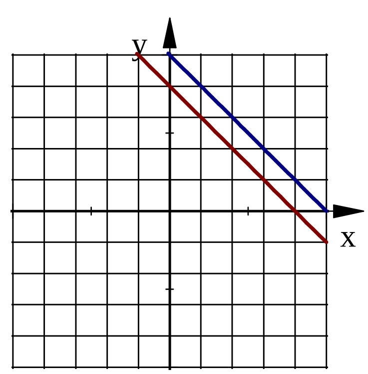


Solution.Set.$=\{(3,1)\}$ or: $x = 3$ and $y = 1$

This system has **exactly one** solution point.

ii. The empty set (the two lines are **parallel**)

$$\begin{cases} x + y = 5 \\ x + y = 4 \end{cases}$$

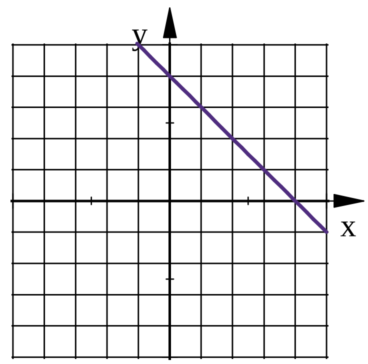


S.S.$=\phi$ [the empty set].

There are **no solutions**.]

iii. The two lines are **colinear** [same line]

$$\begin{cases} 2x + 2y = 8 \\ x + y = 4 \end{cases}$$



S.S.=$\{(x, y) | x + y = 4\}$

There are **infinitely many solutions** and the system is called **consistent** and **dependent**.

B. Methods for solving

i. By **graphing**

Even with the aid of a graphing calculator, this method, in many cases, may give only an **approximate decimal** solution.

ii. By **substitution**

Example:

$$\begin{cases} 2x + 3y = 7 \\ 3x - y = 2 \end{cases}$$

Begin by solving the second equation for y :

$$y = 3x - 2$$

substitute this into the first equation:

$$2x + 3(3x - 2) = 7$$

and solve this for x :

$$x = \frac{13}{11}$$

Then use the substitution equation to find y :

$$y = 3 \bullet \frac{13}{11} - 2 = \frac{17}{11}$$

and write the solution set for the system:

$$S.S. = \left\{\left(\frac{13}{11}, \frac{17}{11}\right)\right\}$$

or

$$x = \frac{13}{11} \text{ and } y = \frac{17}{11}$$

iii. By **multiplication-addition** (also called **elimination** or **cancellation**).

Example:

$$\begin{cases} 5x + 6y = 7 \\ 3x - 4y = 2 \end{cases}$$

Multiply the first equation by 3 and the second by -5 :

$$15x + 18y = 21$$
$$-15x + 20y = -10$$

and add these to obtain

$$38y = 11$$
$$y = \frac{11}{38}$$

This can be substituted into either original equation and a solution for x obtained. But it's easier to "cancel" the y's. Multiply the first equation by 2 and the second by 3 :

$$10x + 12y = 14$$
$$9x - 12y = 6$$

and add to obtain

$$19x = 20$$
$$x = \frac{20}{19}$$

Finally, write the solution set:

$$S.S. = \left\{ \left(\frac{20}{19}, \frac{11}{38} \right) \right\}$$

or

$$x = \frac{20}{19} \text{ and } y = \frac{11}{38}$$

Example What happens with one of these analytic methods when the system is inconsistent or dependent?

$$\begin{cases} 6x + 6y = 7 \\ 3x + 3y = 2 \end{cases}$$

Multiply the second equation by -2 to obtain

$$6x + 6y = 7$$
$$-6x - 6y = -4$$

Adding these gives

$$0 = 3$$

a **false** equation. This tells us that the solution set is the empty set:

$$S.S. = \phi$$

Example

$$\begin{cases} 6x + 6y = 4 \\ 3x + 3y = 2 \end{cases}$$

Multiply the second equation by –2 to obtain

$$\begin{aligned} 6x + 6y &= 4 \\ -6x - 6y &= -4 \end{aligned}$$

Adding these gives

$$0 = 0$$

a **true** equation. This tells us that there are infinitely many solutions, and we write:

$$S.S. = \{(x, y) | 3x + 3y = 2\}$$

Graphically, these two equations produce the same line.