

Examples of Solving Trigonometric Equations

Solving by Factoring

Solve $\tan^2 x + \tan x - 2 = 0$ over the interval $[0, 2\pi)$.

$$\begin{aligned}\tan^2 x + \tan x - 2 &= 0 \\ (\tan x - 1)(\tan x + 2) &= 0 \\ \tan x - 1 = 0 \text{ or } \tan x + 2 &= 0 \\ \tan x = 1 \text{ or } \tan x &= -2\end{aligned}$$

The solutions for $\tan x = 1$ over the interval $[0, 2\pi)$ are $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$.

We will need the help of the calculator to solve $\tan x = -2$. Find $\tan^{-1}(-2) = -1.1071487$ (radian mode here). This solution is in QIV but not in the interval $[0, 2\pi)$. To find all solutions in the correct interval we must add π , the period of the tangent function, and then add 2π .

$$\begin{aligned}\pi - 1.1071487 &= 2.0344 \\ \text{and } 2\pi - 1.1071487 &= 5.1760\end{aligned}$$

Remember that we are using radian mode. The solution set is

$$S.S. = \left\{ \frac{\pi}{4}, \frac{5\pi}{4}, 2.0344, 5.1760 \right\}$$

Solving by Squaring

Example: Solve $\tan x + \sqrt{3} = \sec x$ over the interval $[0, 2\pi)$.

$$\begin{aligned}(\tan x + \sqrt{3})^2 &= \sec^2 x \text{ Square each side} \\ \tan^2 x + 2\sqrt{3} \tan x + 3 &= \sec^2 x \\ \tan^2 x + 2\sqrt{3} \tan x + 3 &= 1 + \tan^2 x \text{ Pythagorean identity} \\ 2\sqrt{3} \tan x &= -2 \text{ Subtract } 3 + \tan^2 x \\ \tan x &= -\frac{2}{2\sqrt{3}} = -\frac{\sqrt{3}}{3} \text{ Divide by } 2\sqrt{3} \text{ and simplify}\end{aligned}$$

The two possible solutions from this last equation are in Quadrants II and IV with a reference angle of $\frac{\pi}{6}$ (radians). This gives *possible* solutions $\frac{5\pi}{6}$ or $\frac{11\pi}{6}$. However, since both sides were squared, **extraneous solutions (roots)** are possible. We must check each proposed solution

$$\begin{aligned}\tan \frac{5\pi}{6} + \sqrt{3} &= \sec \frac{5\pi}{6} ? & \tan \frac{11\pi}{6} + \sqrt{3} &= \sec \frac{11\pi}{6} ? \\ \frac{-\sqrt{3}}{3} + \sqrt{3} &= -\frac{2\sqrt{3}}{3} ? & \frac{-\sqrt{3}}{3} + \sqrt{3} &= \frac{2\sqrt{3}}{3} ? \\ \frac{2\sqrt{3}}{3} &\neq -\frac{2\sqrt{3}}{3} & \frac{2\sqrt{3}}{3} &= \frac{2\sqrt{3}}{3} \\ \frac{5\pi}{6} &\text{ is not a solution} & \frac{11\pi}{6} &\text{ is a solution}\end{aligned}$$

$$S.S. = \left\{ \frac{11\pi}{6} \right\}$$

You can get a visualization of this by graphing

$$Y_1 = \tan X + \sqrt{3} - 1/\cos(X)$$

in the window $[0, 2\pi] \times [-4, 4]$ with $Xscl = \frac{\pi}{6}$ and noting the x -intercept at the 11th scale mark.