Worksheet for Rational Functions

Write the rational function as the quotient of two polynomials, each in standard form.

$$f(x) = \frac{P(x)}{Q(x)} = \frac{a_n x^n + \cdots + a_0}{b_m x^m + \cdots + b_0}$$

1. **Domain**: solve the equation $Q(x) = 0$. Each *real number* solution is a number *excluded* from the domain of the rational function.

   Answer: $x =$ ______________________________

   A. Are there any real solutions? __________ (yes/no) If yes, write the domain below and then go to Part 2 – Intercepts and Holes. If not, skip to part 1B.

   Domain = $\{x \mid x$ is a real number and $x \neq$ $\}$

   B. Are **all** the zeros of $Q(x)$ complex (non-real) numbers? ____________ (yes/no) If yes, then

   Domain = $(-\infty, \infty)$ = the set of all real numbers

2. **Intercepts and Holes**: Solve the equation $P(x) = 0$ for x,. and then list the *real number* zeros of $P(x)$.

   Answer: $x =$ ______________________________

   A. List the numbers you have found that are in the domain of the rational function (they are **not** excluded). Each one of these gives an x – intercept for the graph.

   x – intercept(s) at $x =$ ____ ("**None**" is possible)

   B. List any numbers that appear in **both** lists in Part 1 and Part 2–they are zeros of both $P(x)$ and $Q(x)$. At each of these value(s) of x, there will be a **hole** in the graph.

   The graph has a hole/holes at $x =$________________________ ("**None**" is possible)

   C. Is $x = 0$ in the domain of the rational function (not excluded in Part 1)? ___________ (yes/no) If yes, then $b_0 \neq 0$ and the y – intercept is $y = \dfrac{a_0}{b_0}$.

   y – intercept at $y =$ ______________ ("**None**" is possible)

3. **Vertical asymptotes**: The graph of the rational function will have a vertical asymptote line through each zero of $Q(x)$ that is **not** the location of a hole:

   Vertical asymptotes are at $x =$_______________________________________ (there may be more than one V.A.)

4. **Horizontal Asymptote**: Identify the degree of the numerator and denominator polynomials:

Degree of $P(x) = m =$____________

Degree of $Q(x) = n =$____________

A. Is $m < n$? _____________(yes/no) If yes, then write $y = 0$ as the horizontal asymptote below.

B. Is $m = n$?_____________(yes/no) If yes, write $y = \frac{a_m}{b_n}$ [quotient of leading coefficients; simplify this fraction] as the horizontal asymptote.

C. Is $m > n$? _____________(yes/no) If yes, write "**None**" in the space for horizontal asymptote.

Horizontal Asymptote: ___________________________________

5. **Oblique Asymptote**: Is $m = n + 1$? ______________ (yes/no)

If the answer is no, then the analysis is complete. Write "**None**" in the blank, and you are ready to sketch a graph.

If the answer is yes, then you must use division to re-write the rational function in the form

$$\frac{P(x)}{Q(x)} = ax + b + \frac{R(x)}{Q(x)}$$

The remainder $R(x)$ is a polynomial whose degree is less than n.

The line $y = ax + b$ is the oblique asymptote. Write it here (or "None")

Oblique Asymptote: ____

6. **Graph**: sketch asymptote lines as dotted lines (except the axes), and sketch the graph using an appropriate scale. Here is a zoom standard window.