To Log Or Not To Log

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March 9, 2015

Least Squares Linear Regression

- Used to fit functions to data.
- College Algebra.
- Matrix and Linear Algebra.
- Mutivariable Calculus.
- Numerical Analysis and Approximation Theory.

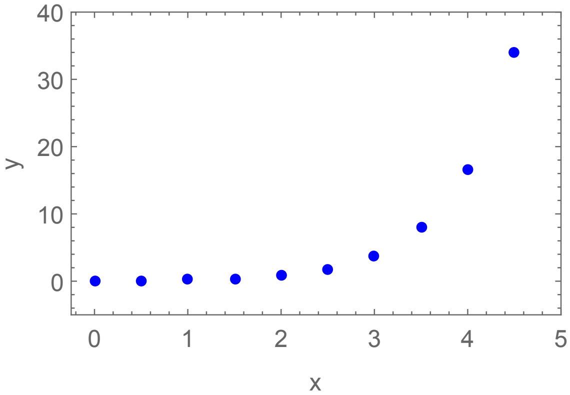
Linearizion of Models

$$y = Ax^r \;\Leftrightarrow\; \ln(y) = r\ln(x) + \ln(A)$$

$$y = Ce^{kx} \;\Leftrightarrow\; \ln(y) = kx + \ln(C)$$

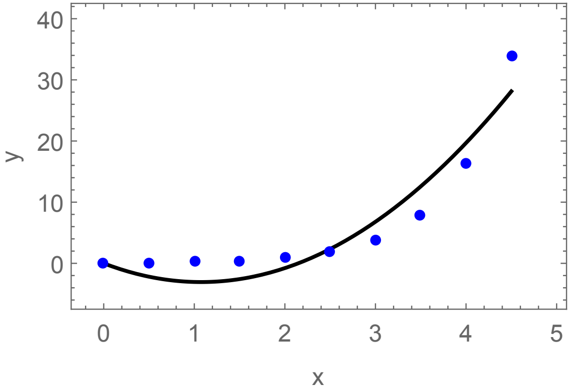
Note: Each model has two parameters.

Data



Note: One of the data points is (0,0).

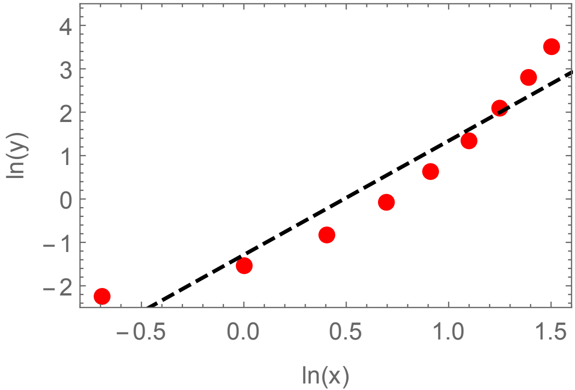
Quadratic Polynomial Fit: $y = ax^2 + bx$



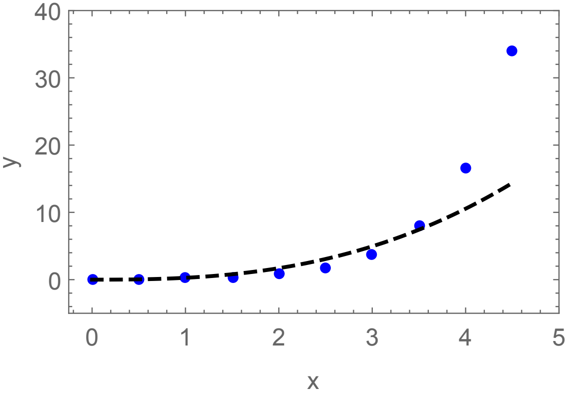
$$y(x) = 2.65626x^2 - 5.70446x$$

$$\sum [y_i - y(x_i)]^2 = 102.428$$

Power Function Fit: $y = Ax^r$



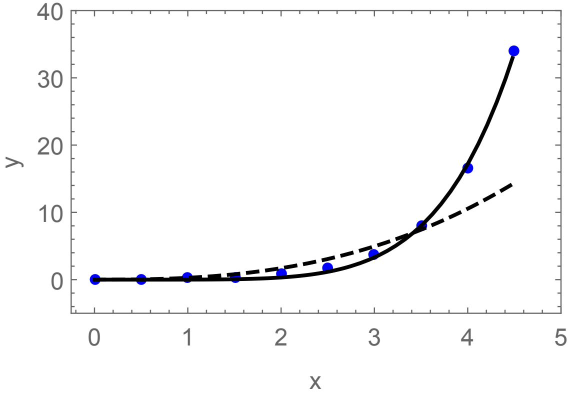
Power Function Fit Using Log-Log (Power-regression in Modern Calculators)



$$y(x) = 0.276344x^{2.62683}$$

$$\sum [y_i - y(x_i)]^2 = 424.794$$

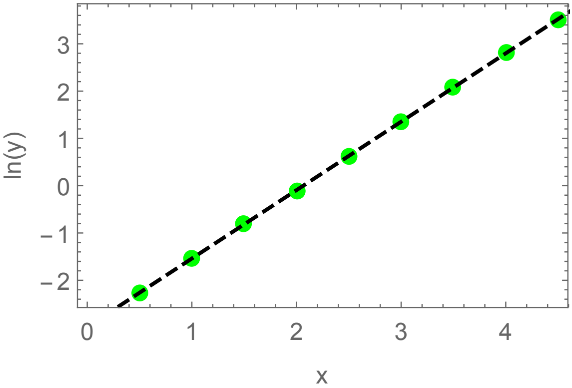
Power Function Fit Using Multivariable Calculus (Levenberg-Marquardt)



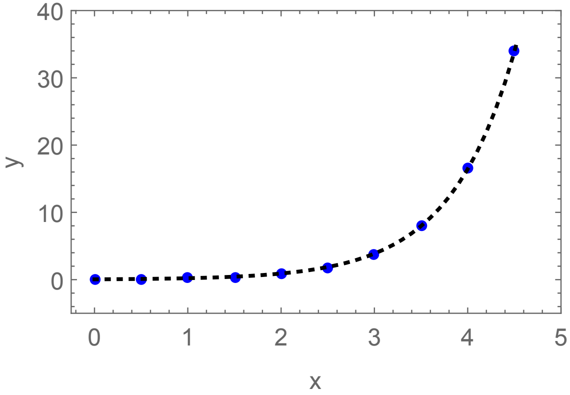
$$y(x) = 0.00612061x^{5.72753}$$

$$\sum [y_i - y(x_i)]^2 = 1.84181$$

Exponential Function Fit: $y = Ce^{kx}$



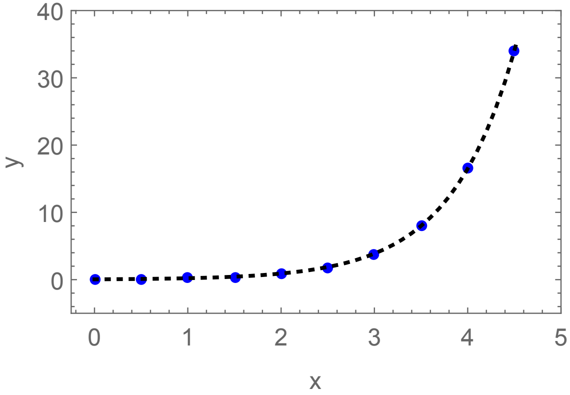
Exponential Function Fit Using Semi-Log (Exponential-regression in Modern Calculators)



$$y(x) = 0.050496e^{1.44618x}$$

$$\sum [y_i - y(x_i)]^2 = 0.0325555$$

Exponential Function Fit Using Multivariable Calculus (Levenberg-Marquardt)



$$y(x) = 0.0501896 e^{1.44856x}$$

$$\sum \left[y_i - y(x_i)\right]^2 = 0.00420048$$

Conclusion

- Linearizing using Log's is easy to teach and implement.
- Using Log's can lead to egregious errors.
- Levenberg-Marquardt method gives much more accurate results for non-linear models and does not require the use of logarithms.

Copies of the talk and implimentation of all methods shown (using *Mathematica*) can be found at http://faculty.tarleton.edu/white/