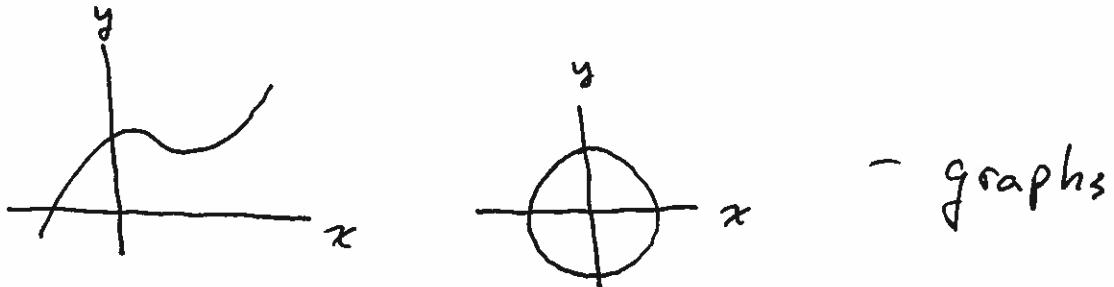


Intro - Review of Calculus I

functions and relations:

$$y = f(x), \quad x^2 + y^2 = 1 \quad - \text{ notation}$$



relation: set of ordered pairs

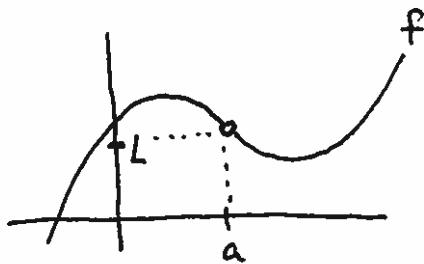
function: relation with vertical line test
(for all x values, there is a unique y value)

Limits

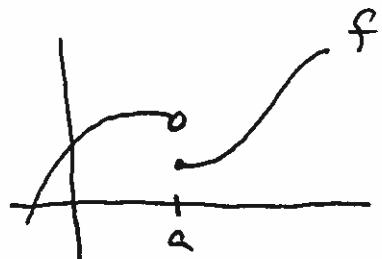
Def $\lim_{x \rightarrow a} f(x) = L \Leftrightarrow$

$$\forall \varepsilon > 0, \exists \delta > 0 \ni 0 < |x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon$$

(\forall = for all, \exists = there exists, \ni = such that)

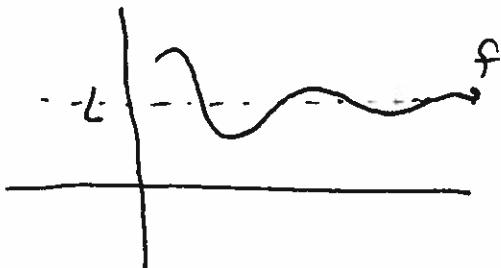


limit exists at a

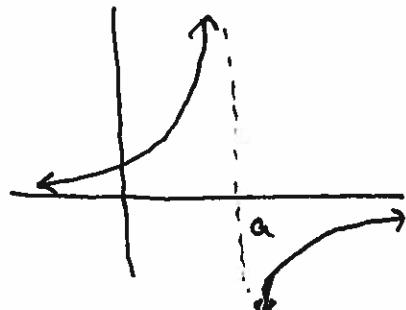


limit DNE at a

Infinite Limits



$$\lim_{x \rightarrow \infty} f(x) = L$$



$$\lim_{x \rightarrow a^-} f(x) = \infty$$

$$\lim_{x \rightarrow a^+} f(x) = -\infty$$

Derivatives

Def $f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$

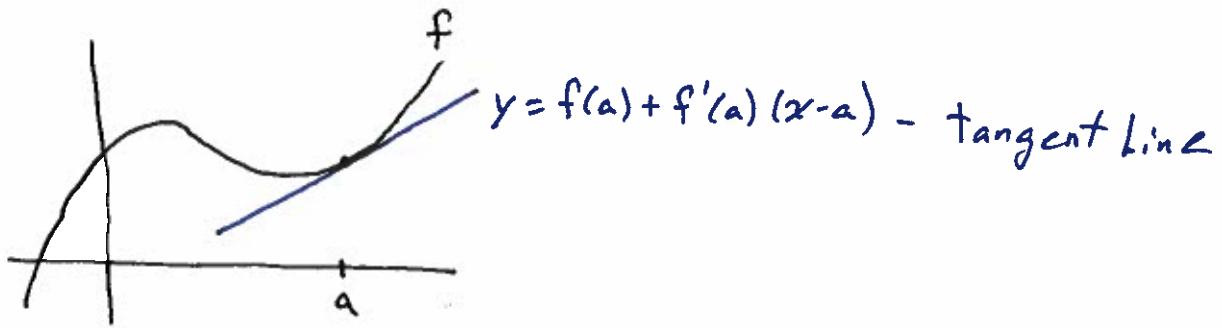
— or —
 $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

Continuity

Def f is continuous at a if

- (a) $f(a)$ exists
- (b) $\lim_{x \rightarrow a} f(x)$ exists
- (c) $\lim_{x \rightarrow a} f(x) = f(a)$

Polynomials, Rational and Radical functions
are continuous on their domains.



Critical points: when $f'(x) = 0$

Local Max/Min, Concavity, points of inflection.

Notations: $f'(x)$, $f''(x)$, $f^{(3)}(x)$

$$\frac{df}{dx}, \frac{d^2f}{dx^2}, \frac{d^3f}{dx^3}$$

1st 2nd 3rd derivatives

Differentials:

$$\Delta y \approx dy = f'(x) dx$$

Integration

Fundamental theorem of Calculus (FTC)

Let F be an antiderivative of f ($F'(x) = f(x)$)
then

$$(1) \quad \int_a^b f(x) dx = F(b) - F(a)$$

$$(2) \quad \frac{d}{dx} \left[\int_c^x f(t) dt \right] = f(x)$$

Limit definition:

$$\int_a^b f(x) dx = \lim_{\|P\| \rightarrow 0} \sum_{k=1}^n f(x_k^*) \Delta x_k$$

provided the limit exists, where

$$P = \{x_0, x_1, x_2, \dots, x_n\}, \quad a = x_0 < x_1 < x_2 < \dots < x_n = b,$$

$$\Delta x_k = x_k - x_{k-1}, \quad \|P\| = \max \{\Delta x_k\},$$

$$x_k^* \in [x_{k-1}, x_k].$$

So provided the integral exists,

$$\int_a^b f(x) dx \approx \sum_{k=1}^n f(x_k^*) \Delta x$$

where $\Delta x = \frac{b-a}{n}$, $x_k = a + k\Delta x$ and

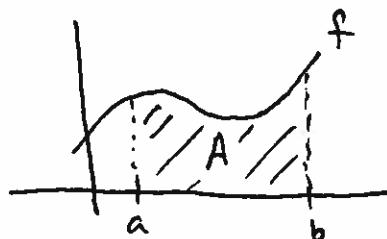
(i) Left end point method: $x_k^* = x_{k-1}$

(ii) Right end point method: $x_k^* = x_k$

(iii) Midpoint method: $x_k^* = \frac{1}{2}(x_{k-1} + x_k)$

$$x_k^* = a + (k - \frac{1}{2})\Delta x$$

Area "under" the curve:



$$A = \int_a^b f(x) dx$$