Chapter 1

Example 2

$$f(x) = \ln(x) \qquad\qquad f(1) = 0$$
$$f'(x) = \frac{1}{x} = x^{-1} \qquad\qquad f'(1) = 1$$
$$f''(x) = -x^{-2} \qquad\qquad f''(1) = -1$$
$$f'''(x) = 2x^{-3} \qquad\qquad f'''(1) = 2$$
$$f^{(4)}(x) = -(3)(2)x^{-3} = -3!\,x^{-4} \qquad\qquad f^{(4)}(1) = -3!$$
$$\vdots \qquad\qquad\qquad \vdots$$
$$f^{(n)}(x) = (-1)^{n-1}(n-1)!\,x^{-n} \qquad\qquad f^{(n)}(1) = (-1)^{n-1}(n-1)!$$

So the Taylor series is

$$T(x) = 0 + 1(x-1) - \frac{1}{2!}(x-1)^2 + \frac{2}{3!}(x-1)^3 - \frac{3!}{4!}(x-1)^4 + \dots$$
$$= (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \dots$$

To find n so that $T_n(x)$ approximates $f(x)$ accurate to 3 decimal digits on $(\frac{1}{2}, \frac{3}{2})$ we need

$$|R_n(x)| < 10^{-3} \quad \text{for } x \in \left(\tfrac{1}{2}, \tfrac{3}{2}\right)$$

$$\Rightarrow \quad |R_n(x)| = \left| \frac{(-1)^n (n!)\, \xi^{-(n+1)}}{(n+1)!} (x-1)^{n+1} \right|$$

where ξ is between x and $a = 1$.

$$|R_n(x)| \le \frac{(\frac{1}{2})^{-(n+1)}}{n+1} \left(\tfrac{1}{2}\right)^{n+1} = \frac{1}{n+1} \overset{\text{set}}{<} 10^{-3}$$

so $n > 999$. That is $n = 1000$

Example 3

$P_3(x) = x - \frac{x^3}{3!}$ (see calc II)

Note that, in this problem, $P_3(x) = P_4(x)$ so $n=4$. $f(x) = \sin(x)$, then $|f^{(n)}(x)| \le 1$ for all x. So

$$|R_4(x)| \le \frac{1}{4!}\left(\frac{\pi}{6}\right)^4 \approx 0.00313$$

Example 9

$$\text{absolute error} = |1{,}005{,}862 - 1{,}000{,}000|$$
$$= 5{,}862 \quad \text{or} \quad 5.8 \times 10^3$$

$$\text{relative error} = \left|\frac{1{,}005{,}862 - 1{,}000{,}000}{1{,}005{,}862}\right|$$

$$\approx 0.005828 \quad \text{or} \quad 5.8 \times 10^{-3}$$

Example 11

$n = 3$, $m = 4$, so $d = 2^2 - 1 = 3$.

$c = 100_2 = 4_{10}$, $f = 1011_2 = 1\cdot\frac{1}{2} + 0\left(\frac{1}{2}\right)^2 + 1\cdot\left(\frac{1}{2}\right)^3 + 1\cdot\left(\frac{1}{2}\right)^4$

So the number 01001011 is

$$(-1)^0\left(2^{4-3}\right)\left(1 + \frac{1}{2} + \frac{1}{8} + \frac{1}{16}\right) = 3.375$$

Example 12

$$\begin{array}{r} 1.08 \\ +\ .00959 \\ \hline 1.08 \end{array}$$ by chopping

then

$$\begin{array}{r} 1.08 \\ +\ 0.00903 \\ \hline 1.08 \end{array}$$ by chopping

so $1.08 + 9.58 \times 10^{-3} + 9.03 \times 10^{-3} = 1.08$

Notice

$$\begin{array}{r} .00903 \\ +\ .00959 \\ \hline .01862 \end{array}$$ so $9.03 \times 10^{-3} + 9.59 \times 10^{-3} = 1.86 \times 10^{-2}$

then

$$\begin{array}{r} .0186 \\ +\ 1.08 \\ \hline 1.0986 \end{array}$$ so

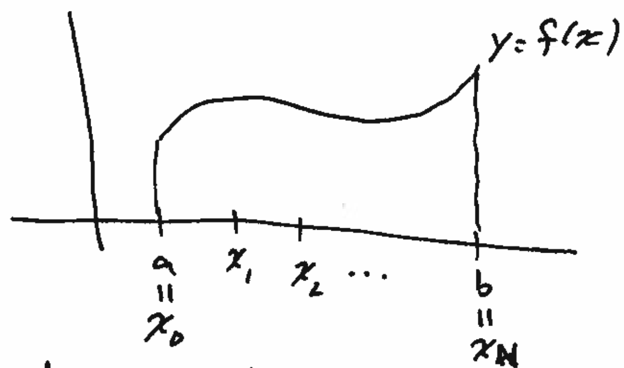
$$9.03 \times 10^{-3} + 9.59 \times 10^{-3} + 1.08 = 1.09 \text{ by chopping}$$

Rounding also has a different outcome depending on the order.

Example 14 Self evident

Example 17

Graphically:



$$\Delta x = \frac{b-a}{N}$$

midpoints

$$x_k^* = a + (k - 1/2)\Delta x$$

then

$$\int_a^b f(x)\,dx = \left(\sum_{k=1}^{n} f(x_k^*)\right)\Delta x$$

Code:

```
set dx = (b-a)/N
Set xstar = a + dx/2
set sum = 0
loop: for j = 1, 2, 3, ..., N
        sum = sum + f(xstar)
        xstar = xstar + dx
end loop
set answer = sum * dx
Output answer
```

Inputs: a, b, N, f